Taylor Polynomials

1. Find the degree 3 Taylor polynomial T_3 of the function $f(x) = \sqrt{x}$ centered at the point $c = 1$.

2. For an arbitrary function $f : \mathbb{R}^2 \to \mathbb{R}$, write

$$\sum_{|\alpha|=3} D^{\alpha}_{\mathbf{c}}(f)$$

   in terms of the partial derivatives of f at the point $\mathbf{c}$.

3. For an arbitrary function $f : \mathbb{R}^3 \to \mathbb{R}$, write the degree 2 Taylor polynomial of f centered at $\mathbf{c}$ in terms of the partial derivatives of f at the point $\mathbf{c}$.

4. Find the degree 2 Taylor polynomial T_2 of the function $f(\begin{bmatrix} x \\ y \\ z \end{bmatrix}) = \dfrac{3+x^2}{y+z}$ centered at the point $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

5. Suppose $\mathcal{M} \subset \mathbb{R}^2$ is the set of points (x,t) which satisfy the following equation

$$F(x,t) = x^3 + xt + t^2 - 7$$

Notice that the point (1,2) is an element of $\mathcal{M}$. The implicit function theorem implies that there are neighborhoods B of 2 and W of 1 and a function $\varphi : B \to W$ such that $(x,t) \in W \times B \in \mathcal{M}$ if and only if $x = \phi(t)$. Find the degree 3 Taylor polynomial of φ centered at 2.

6. Suppose $\mathcal{M} \subset \mathbb{R}^3$ is the set of points (x,s,t) which satisfy the following equation

$$F(x,t) = x^3 + xs + t^2 - 2$$

Notice that the point $(-1,1,2)$ is an element of $\mathcal{M}$. The implicit function theorem implies that there are neighborhoods B of (1,2) and W of 2 and a function $\varphi : B \to W$ such that $(x,s,t) \in W \times B \in \mathcal{M}$ if and only if $x = \phi(s,t)$. Find the degree 2 Taylor polynomial of φ centered at (1,2).