

Quadratic Forms

1. Find a symmetric matrix Q such that the quadratic form

$$Q(x,y) = \frac{x^2}{2} + xy$$

can be written as

$$[x \ y] Q \begin{bmatrix} x \\ y \end{bmatrix}.$$

2. Find a symmetric matrix Q such that the quadratic form

$$Q(x,y) = x_1^2 + 2x_1x_2 + 2x_2^2 + 4x_1x_3 + 5x_2x_3$$

can be written as

$$\vec{x}^t Q \vec{x}$$

with $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

3. Diagonalize the quadratic form in Problem #1 and determine the rank and signature of the quadratic form.
4. Diagonalize the quadratic form in Problem #2 and determine the rank and signature of the quadratic form.
5. Diagonalize the quadratic form $Q(x,y) = xy + 2yz$ and determine the rank and signature of the quadratic form.