Smooth Manifolds

1. Show that the solution set

$$\mathcal{M} = \{(x,y,z) \in \mathbb{R}^3 \mid x^4 - 2y^3 + 3z^2 = 1\}$$

is a smooth manifold. What is the dimension of $\mathcal{M}$?

2. Show that the solution set

$$\mathcal{M} = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 - z = 0, z - y = 3\}$$

is a smooth manifold. What is the dimension of $\mathcal{M}$?

3. Show that the solution set

$$\mathcal{M} = \{(x,y,z) \in \mathbb{R}^3 \mid xy - 8z = 0, x^2 - y = 0, x > 0\}$$

is a smooth manifold. What is the dimension of $\mathcal{M}$?

4. Let $\mathcal{P}$ be the set of positive real numbers $(0,\infty)$. Show that $\mathcal{M} = \gamma(\mathcal{P})$ for the following $\gamma : \mathcal{P} \to \mathbb{R}^2$ is a smooth manifold.

$$\gamma(t) = (t^2 - 1, t(t^2 - 1)).$$

What is the dimension of $\mathcal{M}$?