Linear Algebra

Midterm 1 Spring 2020

Name: ______________________ **ID:** ______________________

Instructions:

(1) Fill in your name and Washington University ID at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(4) Read each questions carefully. Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

(6) This exam has 5 extra credit points.

Problem	1 (25 pts)	2 (30 pts)	3 (30 pts)	4 (20 pts)	Total (105 pts)
Score					

1. In each of the following cases determine whether the given set is a basis for $\mathbb{R}^n$:

(i) $n = 3$ and $S = \left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}.$

(ii) $n = 4$ and $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$

(iii) $n = 3$ and $S = \left\{ \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 9 \end{bmatrix} \right\}$.

2. Suppose $T : \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation given as $T(x) = Ax$ where A is the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & -3 & -4 & -1 \\ 1 & 0 & -1 & -1 \end{bmatrix}$$

(i) Determine all vectors in $\ker(T)$.

(ii) Compute Nullity(T).

(iii) Compute Rank(T).

3. Recall that $M_{2\times 2}(\mathbb{R})$ denotes the vector space of 2×2 matrices. A standard basis for $M_{2\times 2}(\mathbb{R})$ is given as

$$S = \left\{ f_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, f_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, f_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, f_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

(You don't need to show that S is a basis.) Suppose $R : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ is a map defined as

$$R\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(i) Show that R is a linear map.

(ii) Write down the matrices $R(f_i)$ as a linear combination of the matrices in $S = \{f_1, f_2, f_3, f_4\}$.

(iii) Find the 4×4 matrix $[R]_S^S$, which is the matrix representation of R with respect to the basis S. (You should be able to find $[R]_S^S$ using your answer to part (ii).)

4. (i) Give a 1-dimensional subspace of

$$V = \{A \in M_{2\times 2}(\mathbb{R}) \mid A^t = A\}$$

Here A^t denotes the transpose of A. So V is the space of symmetric 2×2 matrices. You may give your answer as multiples of a vector in V. Justify why it is a 1-dimensional subspace.

(ii) Give a 2-dimensional subspace of

$$C(\mathbb{R},\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \mid \ f \text{ is a continuous function}\}.$$

You may give your answer as linear combinations of two elements of $C(\mathbb{R},\mathbb{R})$. Justify why it is a 2-dimensional subspace.