Homework X

1. Use a Taylor polynomial of degree 3 to approximate $\sqrt[5]{33}$.

2. Verify that $y(x)$ satisfies the differential equation, then find a value for the constant C for which $y(x)$ is a solution for the initial value problem. $y(x) = Ce^{-x} + x - 1$, $\begin{cases} y' = x - y \\ y(0) = 5 \end{cases}$

3. Find the solutions of the following initial value problems:

(a) $\begin{cases} y' + \frac{3}{x} y = \frac{\cos(x)}{x^3} \\ y(\pi) = 0 \end{cases}$

(b) $\begin{cases} y' = (1 - y)\cos x \\ y(\pi) = 2 \end{cases}$

(c) $\begin{cases} y' = x^2y^2 - x + x^2 - xy^2 \\ y(2) = 0 \end{cases}$

4. The air in a room with volume 180m^3 contains 0.15% carbon dioxide initially. Fresher air with only 0.05% carbon dioxide flows into the room at a rate of $2\text{m}^3/\text{min}$ and the mixed air flows out at the same rate. Find the percentage of carbon dioxide in the room as a function of time. What happens in the long run?