Homework II

1. Recall that:

$$\sec^2(x) = \tan^2(x) + 1 \qquad \frac{d}{dx}\tan(x) = \sec^2(x) \qquad \frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

   Use u-substitution to evaluate the following indefinite integrals:

   (a) $\int \tan(x)\,dx$

   (b) $\int \sec(x)\,dx$ (Hint: Multiply the fraction by $\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}$ and use substitution with $u = \sec(x) + \tan(x)$.)

   (c) $\int \sec^4(x)\tan^4(x)\,dx$

(d) $\int \sec^5(x)\tan^5(x)\,dx$

(e) $\int x^5(1+x^2)^{\frac{3}{2}}\,dx$

2. In the class, we showed that:

$$\int \cos^2(x)\,dx = \frac{x}{2} + \frac{\sin(2x)}{4} + C \tag{1}$$

We already know how to evaluate the following integral using trigonometric identities. Here we want to solve this integral in a different way. Firstly use integration by parts with $u = \cos^3(x)$ and $dv = \cos(x)dx$. Then use the trigonometric identity $\cos^2(x) + \sin^2(x) = 1$ to write down the integral (2) in terms of the integral of $\cos^2(x)$. Then evaluate the integral using (1).

$$\int \cos^4(x)\,dx \tag{2}$$

3. Evaluate the following integrals:

(a) $\displaystyle\int \ln(\sqrt[10]{x})\,dx$

(b) $\displaystyle\int x^2\cos(4x)\,dx$

(c) $\displaystyle\int \arcsin(x)\,dx$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} \cos(\sqrt{x})\,dx$

(e) $\displaystyle\int_2^{e} \frac{1}{x\ln(x)}\,dx$

(f) $\displaystyle\int \sin^5(x)\cos^{10}(x)\,dx$

(g) $\int \cos^2(5x)\sin(6x)\,dx$

(h) $\int_{-10}^{10} \sin(x)e^{x^2}\,dx$