Homework V

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent:

   (a) $\int_5^\infty xe^{x/5}dx$. (Hint: You can decide based on the shape of the function without finding an anti-derivative for the integrand.)

   (b) $\int_{-\infty}^0 xe^{x/5}dx$.

   (c) $\int_0^3 \frac{e^t}{e^{2t}-1}dt$.[(1)]

[(1)] This is Problem 1.(d) from Homework II. The integral in that problem was unintentionally improper. Now you know enough to solve this problem completely.

(d) $\displaystyle\int_{-\infty}^{\infty} \frac{e^t}{e^{2t}+1}\,dt.$

(e) $\displaystyle\int_{1000}^{\infty} \sin(\theta)\,d\theta.$

(f) $\displaystyle\int_{0}^{10} \frac{1}{(x-5)^2}\,dx.$

(g) $\displaystyle\int_0^{10} \frac{1}{(x-5)^{\frac{1}{3}}}\,dx.$

2. Use the comparison theorem to show that the following integral is convergent:

$$\int_{-\infty}^{\infty} \frac{|\sin x|}{1+x^2}\,dx.$$

3. Use the comparison theorem to show that the integral is divergent.

$$\int_0^1 \frac{e^{x^2}}{x^2}\,dx$$