Homework VI

1. Determine whether the following sequences converge or diverge. Find the limit of those that are convergent:

   (a) $a_n = \tan(\frac{8n\pi}{17+32n})$ for $n \geq 1$.

   (b) $a_n = \frac{e^n + e^{-n}}{e^{2n} + 10}$ for $n \geq 1$.

   (c) $a_n = \cos(\frac{n+1}{4})$ for $n \geq 1$.

(d) $a_n = \cos(\frac{4}{n+1})$ for $n \geq 1$.

2. For each of the following series determine if it is a geometric series. In each case evaluate the series if it is convergent.

(a) $\sum_{n=1}^{\infty} \frac{3 \cdot 2^{n+2}}{5^{n-1}}$.

(b) $\sum_{n=3}^{\infty} \frac{3 \cdot 2^{n+2}}{5^{n-1}}$.

(c) $0.001 + \sqrt{0.001} + \sqrt[3]{0.001} + \ldots + \sqrt[n]{0.001} + \ldots$.

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{\pi^n} - 0.2^n \right)$

3. Write the number $2.3\overline{12} = 2.312121212...$ as the ratio of two integers (in a reduced form).

4. Determine whether the following series converge or diverge:

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+7}{\sqrt{n+1}}$.

(c) $\sum_{n=1}^{\infty} \frac{|\sin(n)|}{1+n^2}$.

(d) $\sum_{n=1}^{\infty} \cos(\frac{4}{n+1})$.