Homework VII

1. Determine whether the following series converge or diverge:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots.$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \arctan(2n).$

(c) $\sum_{n=1}^{\infty}(-1)^n(\frac{\pi}{2} - \arctan(2n))$.

(d) $\sum_{n=1}^{\infty}\frac{\cos(1/n)}{\sqrt{n}}$.

2. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is a convergent series. We use the sum of the first 5 terms to give an approximation to the sum of the series. Then the following series gives the exact value of the remainder term R_5:

$$\sum_{n=6}^{\infty} \frac{1}{n!} = \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \dots \tag{1}$$

(a) For $n \geq 6$, show that:

$$0 \leq \frac{1}{n!} \leq \frac{1}{6!} \cdot (\frac{1}{7})^{n-6}$$

(b) We want to find an upper bound for the remainder term R_5 in (1). Obtain such a bound by comparing the series in (1) with a geometric series and evaluating the geometric series. (Hint: Use part (a).)

3. (a) Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$. We use the sum of the first 10 terms to approximate the sum of this series. Estimate the error involved in this approximation.

(b) How many terms are required to ensure that the sum is accurate to three decimal places.

4. Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. We use the sum of the first 5 terms to approximate the sum of this series. Estimate the error involved in this approximation.