

Homework VIII

1. Determine whether the following series converge or diverge:

$$(a) \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}.$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{n^7 7^n}{n!}.$$

$$(c) \sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}.$$

$$(d) \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}.$$

2. For a series $\sum_{n=1}^{\infty} a_n$ we have $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{2}$.

(a) What can you say about the convergence of the series $\sum_{n=1}^{\infty} na_n$. Justify your answer.

(b) How about the series $\sum_{n=1}^{\infty} n3^n a_n$?

3. In each case, determine the series is absolutely convergent, conditionally convergent or divergent.

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{1}{n \ln(n)}.$$

$$(b) \sum_{n=1}^{\infty} \frac{(-3)^n}{n^n}.$$

4. Determine the interval of convergence of the following series:

$$(a) \sum_{n=1}^{\infty} \frac{(x-2)^n}{n^3}$$

$$(b) \sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2}$$

$$(c) \sum_{n=1}^{\infty} \frac{(1-2x)^n}{n}.$$

$$(d) \sum_{n=1}^{\infty} \frac{(x+1)^n}{2^{n^2}}.$$