Calculus II

Final Exam Fall 2018

Name: ______________________ **ID:** ______________________

Instructions:

(1) Fill in your name and Columbia University ID at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

Problem	1 (16 pts)	2 (16 pts)	3 (16 pts)	4 (20 pts)	5 (16 pts)	6 (16 pts)	7 (24 pts)	8 (24 pts)	Total (148 pts)
Score									

1. Compute the following integrals:

   (a) $$\int \frac{10x - 19}{x^2 - x - 12}\, dx$$

   (b) $$\int_1^e \frac{(\ln(x))^2 + 1}{x}\, dx$$

2. A large tank contains 50 kg of salt dissolved in 10000 liters of water. A solution of water and salt containing 0.02 kg of salt per liter enters the tank through a pipe at a rate of 3 liters per minute. At the same time, the mixture of salt and water is pumped out of the tank at a rate of 3 liters per minute. The tank is kept well mixed and the concentration of salt in the tank is uniform. Let $S(t)$ denote the amount of salt in the tank after t minutes, measured in kg .

   (a) Write down a differential equation for $S(t)$.

   (b) Write down an initial condition for $S(t)$.

   (c) Solve the differential equation with the initial condition given in parts (a) and (b).

3. (a) Find the interval of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{5^n (x+1)^n}{\sqrt{n+4}}$$

(b) Find the Maclaurin series of $f(x) = xe^{\frac{1}{2}x^4}$. What is the radius of convergence?

4. (I) Which one of the following options gives a convergent improper integral?

(a) $\displaystyle\int_0^1 \frac{1}{x+1}\,dx$

(b) $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$

(c) $\displaystyle\int_1^\infty \frac{1}{x^2+1}\,dx$

(d) $\displaystyle\int_1^\infty \frac{1}{x+1}\,dx$

(II) We are looking for the Maclaurin series of a function $f(x)$. We know that $f^{(20)}(0) = 19! \cdot 2^{19}$. What is the coefficient of x^{20} in the Maclaurin series of $f(x)$?

(a) $20! \cdot 19! \cdot 2^{19}$ (b) $\dfrac{1}{20 \cdot 2}$ (c) $\dfrac{2^{19}}{20}$ (d) $19! \cdot 2^{19}$

Recall that the coefficient of x^n in a power series $\displaystyle\sum_{i=0}^{\infty} a_i x^i$ defined to be a_n.

(III) A power series is convergent at -1 and divergent at 1. Which one of the following points could be in the interval of the convergence of this power series:

(a) 2 (b) -2 (c) 4 (d) 10

(IV) Write down a convergent **geometric** series with infinitely many non-zero terms such that the sum is equal to e.

5. We want to use a Taylor polynomial of $f(x) = \sqrt[3]{x}$ to approximate $\sqrt[3]{25}$.

   (a) What would be a good choice for the center of the Taylor polynomial? Why?

   (b) Use the Taylor polynomial of degree 2 centered at the point that you found in the previous part to write an estimate for $\sqrt[3]{25}$.

   (c) Find an upper bound for the error of your estimate.

6. (a) Solve the following initial value problem:

$$xy' + y^2 = 2x^2y^2 \qquad\qquad y(1) = 0$$

(b) Find the general solution of the following differential equation:

$$(x^2+1)y' + 3x^3y = 6xe^{-\frac{3}{2}x^2}$$

7. (a) Determine whether the following series is convergent or divergent. If it converges, find its sum.

$$\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n)!3^{2n}}$$

(b) Determine whether the following series is convergent or divergent. State the test you use, and show your work:

$$\sum_{n=2}^{\infty} \frac{2+\cos(n^2)}{\sqrt{n}-1}$$

(c) We wish to approximate the sum of the following series with error less than 0.0001.

$$\sum_{n=1}^{\infty} \frac{1}{n^5}$$

Determine a partial sum of this series which gives such an approximation. Justify your answer.

8. (a) Let $\mathcal{R}$ be the region enclosed by the x-axis, $x = 2$ and $y = x^2$. Find the volume obtained by rotating $\mathcal{R}$ about the line $y = -4$.

(b) For $\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$, let $g(t) = \displaystyle\int_{\frac{\pi}{2}}^{t} \sqrt{\tan^2(x) - 1}\, dx$. Find the length of the graph of this function for $\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$.

(c) Find the area of the surface obtained by rotating the curve $9x = y^2 + 18$, $2 \le x \le 6$, about the x axis.