Calculus II
Midterm 2 Fall 2018

Name: ______________________ **ID:** ______________________

Instructions:

(1) Fill in your name and Columbia University ID at the top of this cover sheet.

(2) This exam is closed-book and closed-notes; no calculators, no phones.

(3) Leave your answers in exact form (e.g. $\sqrt{2}$, not ≈ 1.4) and simplify them as much as possible (e.g. $1/2$, not $2/4$) to receive full credit.

(4) Answer all questions in the space provided. If you need more room use the blank backs of the pages.

(5) Show your work; correct answers alone will receive only partial credit.

(6) This exam has 5 extra credit points.

Problem	1 (10 pts)	2 (10 pts)	3 (10 pts)	4 (10 pts)	5 (15 pts)	6 (15 pts)	7 (15 pts)	8 (20 pts)	Total (105 pts)
Score									

1. (I) (2 points) Which one of the following options is correct?

(a) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ are both convergent.

(b) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ are both divergent.

(c) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is convergent and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(d) The sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ is divergent and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent.

(II) (2 points) Of the following series listed below, select ALL which are geometric series. (There are exactly two correct answers.)

(a) $\sum_{n=1}^{\infty} \frac{e^n}{n}$ (b) $\sum_{n=1}^{\infty} 2^{\frac{1}{n}}$ (c) $\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^{n-1}}$ (d) $\sum_{n=1}^{\infty} n^{\frac{1}{2}}$ (e) $\sum_{n=1}^{\infty} e^{2n+5}$

(III) (3 points) Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ such that $\sum_{n=1}^{\infty} a_n^2$ is convergent. Explain briefly why your series satisfies these two conditions.

(IV) (3 points) Write the number $3.\overline{48} = 3.484848\ldots$ as the ratio of two integer numbers in a reduced form. (Just give a fraction as the final answer. You do not need to justify your answer.)

Determine whether the following three series converge or not. State the tests that you are using and show your work. (10 points for each series)

2. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$

3. $\sum_{n=1}^{\infty} \frac{3n^3}{2n^3 + 3n - 1}$

4. $\sum_{n=1}^{\infty} \sqrt{\frac{n+2}{n^4}}$

5. (15 points) For each of the following improper integrals, determine whether it is convergent or not. If it is convergent, evaluate the integral:

(a) $\int_1^\infty xe^{-x^2}\,dx$

(b) $\int_0^1 \frac{2+\sin(x)}{x^3}\,dx$

6. (a) (5 points) Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^3}$. We use the sum of the first 10 terms to approximate the sum of this series. Estimate the error involved in this approximation.

(b) (5 points) How many terms are required to ensure that the sum is accurate to three decimal places.

(c) (5 points) Consider the series $\sum_{n=1}^{\infty}\frac{1}{n^3}$. We use the sum of the first 5 terms to approximate the sum of this series. Estimate the error involved in this approximation.

7. (a) (7 points) Find the length of the curve $y = \int_0^x \sqrt{t^2 + 6t + 8}\, dt$ for $1 \leq x \leq 4$.

(b) (8 points) Find the area of the surface obtained by rotating the curve $x = 2 + 3y^2$, for $2 \leq y \leq 3$ about the x-axis.

8. (20 points) A swimming pool, filled with water, has the shape of an inverted frustum. (A frustum is obtained from a right circular cone by cutting off the tip.) The radius of the upper and lower bases are respectively equal to 12m and 8m. The height of the pool is also equal to 2m. Find the work required to empty the pool by pumping all of the water to the top of the pool. (The density of water is 1000kg/m^3.)