Sequences and Series

1. Suppose $f : [0,\infty) \to \mathbb{R}$ is a function which is a constant function on each interval of the form $[i,i+1)$ and its value on this interval is equal to a_i.

   (a) For each positive integer n evaluate the integral $\int_0^n f(x)\,dx$.

   (b) When does the improper integral $\int_0^n f(x)\,dx$ exist? Interpret this improper integral as the sum of infinitely many numbers.

2. Does the sequence $a_n = \frac{n}{n^2+1}$ converge? If so, what is the limit?

3. Does the sequence $a_n = \frac{\ln(n)}{n}$ converge? If so, what is the limit?

4. Does the sequence $a_n = \sin(\frac{1}{n}) + \cos(\frac{\ln(n)}{n})$ converge? If so, what is the limit?

5. Determine whether the series $\sum_{i=1}^{\infty} \frac{1}{2^i}$ is convergent or divergent? If it is convergent, what is the sum?

6. More generally, determine when the series $\sum_{i=0}^{\infty} ar^i$ is convergent and evaluate the series?

7. Write the number $2.\overline{08}$ as the ratio of two positive integers.

8. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent or divergent? If it is convergent what is the sum?