Power Series

1. Determine all values of x such that the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n!}$ is convergent.

2. Determine all values of x such that the power series $\displaystyle\sum_{n=1}^{\infty} n!x^n$ is convergent.

3. Determine all values of x such that the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{3^n}(x+1)^n$ is convergent.

4. Determine all values of x such that the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ is convergent.

5. (a) Express $\dfrac{1}{1+x}$ as the sum of a power series and find the interval of convergence.

(b) Express $\dfrac{1}{1+x^3}$ as the sum of a power series and find the interval of convergence.

(c) Express $\dfrac{1}{2+x^3}$ as the sum of a power series and find the interval of convergence.

(d) Express $\dfrac{x^2}{2+x^3}$ as the sum of a power series and find the interval of convergence.

6. Express $\dfrac{1}{(1-x)^2}$ as the sum of a power series and find the interval of convergence.

7. (a) Write a power series which is equal to the derivative of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

   (b) Show that $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is equal to e^x.

8. Find a power series representation for $\ln(1+x)$.