Taylor and Maclaurin Series

1. Find the sum of the power series $\sum_{i=1}^{\infty}(-1)^{n-1}\frac{1}{n\cdot 2^n}$

2. (a) Find the Maclaurin series of the function e^x.

   (b) Find the Taylor series of the function e^x centered at 1.

3. (a) Find the Maclaurin series of the function $\sin(x)$.

   (b) Show that the Maclaurin series of $\sin(x)$ is a power series representation for $\sin(x)$ which is valid for all values of x.

4. Find the Maclaurin series of the function $\cos(x)$. Is it a power series representation for $\cos(x)$?

5. (a) Find a power series representation of $f(x) = x\sin(x)$.

   (b) What is $f^{(31)}(0)$? How about $f^{(32)}(0)$?

6. (a) Find a power series representation of $f(x) = \cos(x^6)$.

(b) What is $f^{(100)}(0)$?

(c) What is $f^{(96)}(0)$?

7. (a) Approximate $\sqrt[3]{7}$ using a Taylor polynomial of degree 2 centered at $x = 8$ for the function $f(x) = \sqrt[3]{x}$.

(b) How accurate is this approximation?

8. (a) What is the maximum error possible in using the Taylor polynomial of degree 5 centered at 0 in approximating the function $\sin(x)$ when $-0.3 \leq x \leq 0.3$? Use this approximation to find $\sin(0.1)$ correct to six decimal places.

(b) For what values of x is this approximation accurate to within 0.00005?