Differential Equations and Modeling

1. A population of bacteria reproduces at a rate proportional to the size of the population. Write down a differential equation which models the population of bacteria. What is the general solution of your differential equation?

2. Write down a differential equation which models fish population in a pond which has carrying capacity of 1000 fish.

3. Write down a differential equation which models the motion of a spring.

4. Chef Cambell is mixing up a large batch of fruit juice ambrosia. Suppose she has a big mixing vat containing 100 gallons of orange juice. At time $t = 0$, she starts pouring a mango / strawberry mixture containing 50% mango juice and 50% strawberry juice into the vat at a rate of 4 gallons per minute. At the same time, mixed tri-flavor juice starts leaving the vat a rate of 4 gallons per minute.

   (a) At time t, how many gallons of tri-flavor juice are there in the vat? (This is asking just for the total amount of stuff in the batch at time t.)

   (b) At time t, what is the rate at which strawberry juice is entering the vat?

   (c) At time t, what is the concentration of strawberry juice in the vat?

   (d) At time t, what is the rate at which strawberry juice is leaving the vat?

   (e) Write a differential equation for the amount $S(t)$ of strawberry juice (measured in gallons) in the vat at time t, where t is measured in minutes. Can you write down an initial condition as well?