

First-Order Linear Differential Equations

1. Solve the following differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = 1$$

2. Solve the following differential equation:

$$\begin{cases} \frac{dy}{dt} + 3t^2y = 6t^2 \\ y(0) = 3 \end{cases}$$

3. Solve the following initial value problem:

$$\frac{dy}{dt} - y = e^{2t}$$

4. Chef Cambell is mixing up a large batch of fruit juice ambrosia. Suppose she has a big mixing vat containing 100 gallons of orange juice. At time $t = 0$, she starts pouring a mango / strawberry mixture containing 50% mango juice and 50% strawberry juice into the vat at a rate of 4 gallons per minute. At the same time, mixed tri-flavor juice starts leaving the vat a rate of 5 gallons per minute.

- (a) At time t , how many gallons of tri-flavor juice are there in the vat? (This is asking just for the total amount of stuff in the batch at time t .)
- (b) At time t , what is the rate at which strawberry juice is entering the vat?
- (c) At time t , what is the concentration of strawberry juice in the vat?
- (d) At time t , what is the rate at which strawberry juice is leaving the vat?
- (e) Write a differential equation for the amount $S(t)$ of strawberry juice (measured in gallons) in the vat at time t , where t is measured in minutes. Can you write down an initial condition as well? Solve the differential equation.

5. Solve the following differential equation:

$$\frac{dy}{dt} + \frac{3t}{t^2 + 1}y = \frac{6t}{t^2 + 1}$$