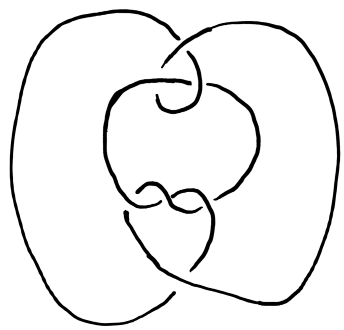
Problem Set 1

1. (a) Let K be a link in S^3. Show that $S^3\backslash K$, the complement of K in S^3, is an invariant of the the equivalence class of K, i.e., two equivalent links have homeomorphic complements.

   (b) Show that complement of the Hopf link is homeomorphic to $S^1 \times S^1 \times (0{,}1)$.

2. Find a sequence of Reidemeister moves and their inverses which relate the following diagram to the standard diagram for the unknot.



3. Let D be a diagram for a link L. An arc of D from one under-crossing to the next one is called a segment of D. A *Fox coloring* of D is a coloring of the segments of D with three colors red, green and blue such that at each crossing either all three colors appear or only one color appears. Show that the number of Fox colorings does not change by applying a Reidemeister move to the diagram D and hence it is an invariant of (the equivalence class of) L. Use this invariant to show that the trefoil knot is not equivalent to the unknot and the figure-eight knot.