Problem Set 10

1. For any integer n, compute Conway polynomials of the link $T_{2,2n}$ for all different possible orientations of the two components.

2. Suppose K is an oriented knot and A is a Seifert matrix for K.

   (a) Show that the matrix $A + A^t$ is invertible.

   (b) Since $A + A^t$ is an invertible symmetric matrix with integer entries, all of its eigenvalues are positive or negative. Show that the number of positive eigenvalues minus the number of negative eigenvalues is an invariant of K and does not depends on the choice of the Seifert matrix.

3. Suppose D is a diagram for an oriented link K and B is a 2-dimensional ball that intersects the diagram in exactly 4 points. We remove $D \cap B$, rotate the diagram 180 degrees about an axis such that the 4 intersection points are preserved and glue it back to the diagram to form the digram of a new link. This operation is called *mutation*. Show that mutation does not change the Conway polynomial of K.

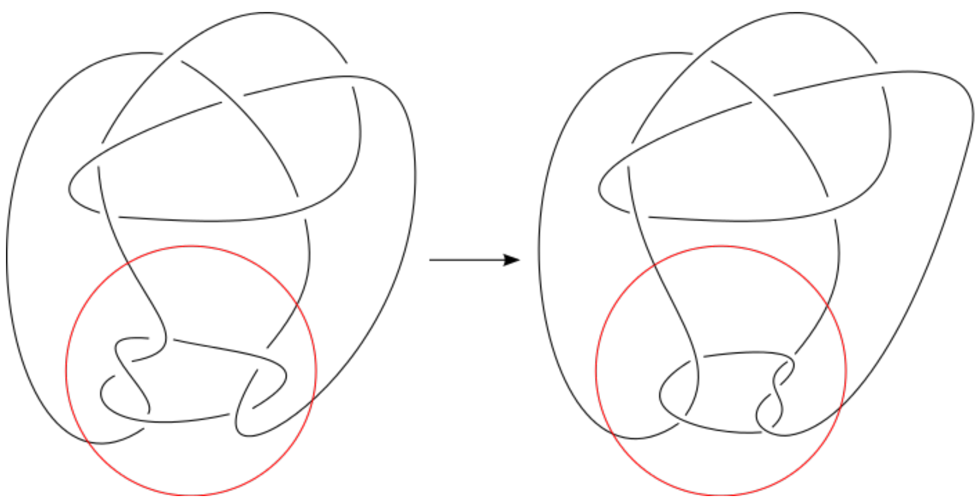

Figure 1: The knot on the left is a mutation of the knot on the right