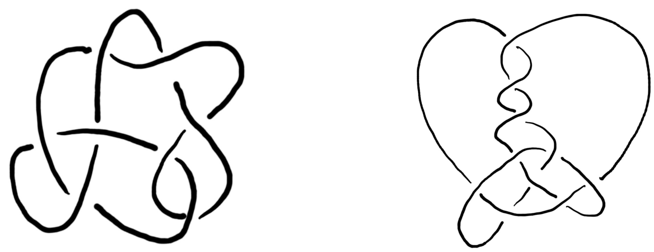
Problem Set 3

1. It is a conjecture that Jones polynomial detects the unknot. That is to say, if a knot K has the same Jones polynomial as the unknot, then K has to be the unknot. However, there are distinct knots with the same Jone polynomials: show that the Jones polynomials of the knots in the following figure are equal to each other.



(a) The knot 8_8 (b) The knot 10_{129}

Figure 1: Two knots with the same Jones Polynomials

2. (a) Suppose $L = L_1 \cup L_2$ is an oriented link with two connected components L_1 and L_2, and $D = D_1 \cup D_2$ is a diagram for L with D_i being a diagram for L_i. We define the *linking number* of L_1 and L_2, denoted by $\mathrm{lk}(L_1, L_2)$ to be the half of the sum of the signs of the crossings of D which have one strand from D_1 and one strand from D_2. (Thus this definition is similar to the definition of writhe except that we only consider signs of some of the crossings, not all of them.) Show that $\mathrm{lk}(L_1, L_2)$ is independent of the choice of the diagram. Why is $\mathrm{lk}(L_1, L_2)$ always an integer?

   (b) Suppose that L' is the oriented link obtained from $L = L_1 \cup L_2$ by changing the orientation of L_1. Show that $J(L') = A^{12\mathrm{lk}(L_1,L_2)} J(L)$.

3. Show that the crossing number of the connected sum of two alternating knots K_1 and K_2 is equal to the sum of the crossing numbers of K_1 and K_2.