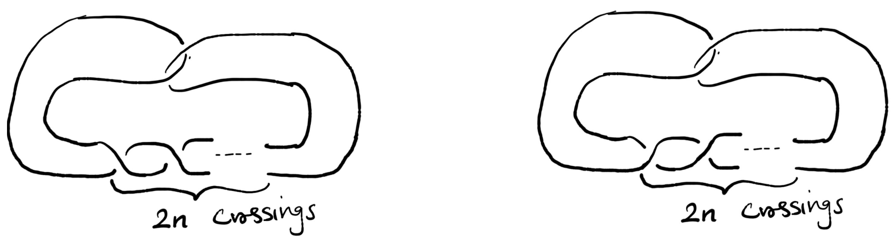
Problem Set 5

1. Suppose K_n is the twist knot given in Figure 1a for positive integers n and in Figure 1b for negative integers n.



(a) K_n for positive n (b) K_n for negative n

Figure 1: Twist knots

   (a) Identify twist knots for $n = 0$, 1 and -1 in terms of the knots that we already introduced in the class.

   (b) Give a presentation of the knot group of K_n, which has two generators and one relation.

2. In Problem Set 1, we defined the notion of Fox coloring for any link digram D, and you showed that the number of Fox colorings is a link invariant. Show that this number is equal to the number of homomorphisms $\phi : \pi_1(X(L)) \to S_3$ such that the conjugacy class of the meridian in $\pi_1(X(L))$ are mapped to permutations that swap two elements of $\{1, 2, 3\}$. (Recall that S_3 is the the symmetric group associated to the set $\{1, 2, 3\}$.)

3. Let G be the knot group of the figure eight knot. By considering homomorphisms from G into a finite group, show that G is not isomorphic to the knot group of the unknot.

4. In the class, we obtained two different presentations of the knot group of the trefoil $T_{2,3}$:

$$\langle x, y \mid xyx = yxy\rangle \qquad \langle a, b \mid a^2 = b^3\rangle$$

   (a) Show directly that these two groups are isomorphic.

   (b) For any knot K, Show that the knot group of its reflection $\overline{K}$ is isomorphic to the knot group of K.

   (c) Show that the connected sum knots $T_{2,3}\#T_{2,3}$ (granny knot) and $T_{2,3}\#\overline{T_{2,3}}$ (square knot) have isomorphic fundamental groups.

   (d) Compute the Jones polynomials of the granny knot and the square knot and conclude that these two knots are not equivalent. This example shows that the prime condition is necessary for Theorem 1.2 in Lecture 7.