## Problem Set 6

1. Compute the homology groups of the surface $\Sigma_{g,d}$ of genus $g$ with $d$ boundary components.

2. Let $\Delta$ be the triangle given in Figure 1. We also assume that each edge of $\Delta$ is identified with the interval $[0, 1]$ such that the orientation of the edge is given by moving along the edge from $0$ to $1$.

   (a) Show that $\Sigma_{g,1}$ admits an oriented triangulation. An oriented triangulation is given by a finitely many homeomorphisms $\{f_i : \Delta \to \Sigma_{g,1}\}_i$ such that

   (i) $\Sigma_{g,1} = \bigcup_i f_i(\Delta)$;

   (ii) For any $i$, the restriction of $f_i$ to the interior of $\Delta$ is disjoint from the image of the remaining homeomorphisms. The restriction of $f_i$ to the interior of each edge $e_r$ of $\Delta$ is either disjoint from the image of the remaining homeomorphisms or there is exactly one other homeomorphism $j$ and one edge $e_s$ of $\Delta$ such that $f_i|_{e_r}(t) = f_j|_{e_s}(1 - t)$.

   (b) Suppose $X$ is a topological space and $\gamma : S^1 \to X$. (Regard $\gamma$ as a map from $[0, 1]$ to $X$ such that $\gamma(0) = \gamma(1)$.) Suppose there is a map $\Gamma : \Sigma_{g,1} \to X$ such that $\Gamma|_{\partial\Sigma_{g,1}} = \gamma$. Show that $\gamma$ represents the trivial element of $H_1(X)$. (Hint: Start from the definition of $H_1(X)$ and show that $\gamma$ belongs to the image of the map $\partial_2 : C_2(X) \to C_1(X)$.)

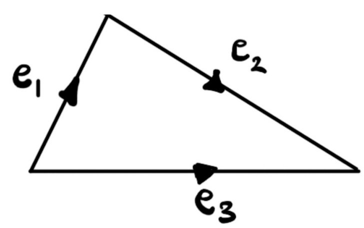


Figure 1: The triangle $\Delta$