Problem Set 7

1. Show that the Seifert genus of the torus knot $T_{p,q}$ is less than or equal to $\frac{(p-1)(q-1)}{2}$.

2. Suppose K is a knot and $N(K)$ is a neighborhood of K which is identified with $S^1 \times D^2$. We remove this regular neighborhood and replace it with the solid torus together with the knot which are sketched in Figure 1.This determines a new knot which is called the *Whitehead double* of K. Show that the Seifert genus of the Whitehead double of any knot K is at most 1.

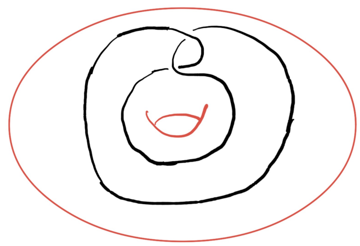

Figure 1

3. (a) In this problem, we identify the 2-dimensional torus $S^1 \times S^1$ with $\mathbb{R}/\mathbb{Z} \times \mathbb{R}/\mathbb{Z} = \mathbb{R}^2/\mathbb{Z}^2$. Using this description of the 2-dimensional torus, show that any element of $GL(2,\mathbb{Z})$ induces a homeomorphism of $S^1 \times S^1$, where $GL(2,\mathbb{Z})$ is the group of invertible 2×2 matrices with integer entires.

   (b) Suppose K_1 and K_2 are two knots in S^3, and $X(K_1)$ and $X(K_2)$ denote the complements of these two knots. We identify the boundaries of $X(K_1)$ and $X(K_2)$ with $S^1 \times S^1$, and use an element of $GL(2,\mathbb{Z})$ to glue these knot exteriors along their boundaries. Explain why the resulting space is a compact 3-manifold without boundary. Determine when the resulting space has trivial H_1.