Problem Set 9

1. Show that the Alexander polynomial of the Whitehead double of any knot K (Problem 2 in Problem Set 7) is equal to 1.

2. (a) Show that the genus of the twist knot K_n from Problem 1 in Problem Set 5 is at most 1.

   (b) Compute the Alexander polynomial of the twist knot K_n.

3. Characterize all possible Alexander polynomials for a knot with genus 1.

4. Suppose K is a knot such that the exterior $X(K)$ is given as follows. There is a homeomorphism f of the surface $\Sigma_{g,1}$ which is identity in a neighborhood of the boundary and:

$$X(K) \cong \{(x,t) \in \Sigma_{g,1} \times [0,1] \mid (x,1) \sim (f(x),0)\}.$$

   Note that any subspace of the form $\Sigma_{g,1} \times \{t\}$ gives a Seifert surface for K. Any such knot is called a fibered knot. For example, trefoil is a fibered knot. (You don't need to show this.)

   (a) The homeomorphism induces an isomorphism f_* from the abelian group $H_1(\Sigma_{g,1}) \cong \mathbb{Z}^{2g}$ to itself. Characterize the Alexander polynomial of K in terms of f_*.

   (b) Show that the genus of K is equal to g.

   (c) Show that the coefficient of the largest (and hence the smallest) power of t in $\Delta_K(t)$ is ± 1.

   (d) Show that the twist knot K_n, for $n \neq -1, 0, 1$, is not fibered.