HOMEWORK 1

MATH 5052, SPRING 2013
DUE WEDNESDAY, JANUARY 23

Exercise 1. Finish Proposition 8.2 by proving that $\|f_k - g_0\|_{(N,\alpha)} \to 0$.

Exercise 2 (Folland, Exercise 8.3)**.** Let $\eta(t) = e^{-1/t}\chi_{(0,\infty)}$.

- **a.** For $k \in \mathbb{N}$ and $t > 0$, $\eta^{(k)}(t) = P_k(1/t)e^{-1/t}$ where P_k is a polynomial of degree $2k$.
- **b.** $\eta^{(k)}(0)$ exists and equals zero for all $k \in \mathbb{N}$.

Exercise 3. Prove that a function f is uniformly continuous iff $\|\tau_y f - f\|_u \to 0$ as $y \to 0$. (This assertion appears at the top of p. 238 in Folland.)

Exercise 4 (Folland, Exercise 8.4)**.** If $f \in L^\infty$ and $\|\tau_y f - f\|_\infty \to 0$ as $y \to 0$, then f agrees a.e. with a uniformly continuous function. (*Hint*: Let $A_r f$ be as in Theorem 3.18. Show that $A_r f$ is uniformly continuous for $r > 0$ and uniformly Cauchy as $r \to 0$.)

Exercise 5 (Folland, Exercise 8.7)**.** If f is locally integrable on $\mathbb{R}^n$ and $g \in C^k$ has compact support, then $f * g \in C^k$.