7. \(a \cdot b = (2i + j) \cdot (i - j + k) = (2)(1) + (1)(-1) + (0)(1) = 1\)

10. By Theorem 3, \(a \cdot b = |a| |b| \cos \theta = (80)(50) \cos \frac{3\pi}{4} = 4000 \left(-\frac{\sqrt{2}}{2}\right) = -2000\sqrt{2} \approx -2828.43\).

20. \(|a| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9, |b| = \sqrt{0^2 + 4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}\), and \(a \cdot b = (8)(0) + (-1)(4) + (4)(2) = 4\).

Then \(\cos \theta = \frac{a \cdot b}{|a| |b|} = \frac{4}{9 \cdot 2\sqrt{5}} = \frac{2}{9\sqrt{5}}\) and \(\theta = \cos^{-1}\left(\frac{2}{9\sqrt{5}}\right) \approx 84^\circ\).

23. (a) \(a \cdot b = (9)(-2) + (3)(6) = 0\), so \(a\) and \(b\) are orthogonal (and not parallel).

(b) \(a \cdot b = (4)(3) + (5)(-1) + (-2)(5) = -3 \neq 0\), so \(a\) and \(b\) are not orthogonal. Also, since \(a\) is not a scalar multiple of \(b\), \(a\) and \(b\) are not parallel.

(c) \(a \cdot b = (-8)(6) + (12)(-9) + (4)(-3) = -168 \neq 0\), so \(a\) and \(b\) are not orthogonal. Because \(a = -\frac{1}{3}b\), \(a\) and \(b\) are parallel.

(d) \(a \cdot b = (3)(5) + (-1)(9) + (3)(-2) = 0\), so \(a\) and \(b\) are orthogonal (and not parallel).

24. (a) \(u \cdot v = (-5)(3) + (4)(4) + (2)(-1) = 3 \neq 0\), so \(u\) and \(v\) are not orthogonal. Also, \(u\) is not a scalar multiple of \(v\), so \(u\) and \(v\) are not parallel.

(b) \(u \cdot v = (9)(-6) + (-6)(4) + (3)(-2) = -84 \neq 0\), so \(u\) and \(v\) are not orthogonal. Because \(u = -\frac{1}{3}v\), \(u\) and \(v\) are parallel.

(c) \(u \cdot v = (c)(c) + (c)(0) + (c)(-c) = c^2 + 0 - c^2 = 0\), so \(u\) and \(v\) are orthogonal (and not parallel). (Note that if \(c = 0\) then \(u = v = 0\), and the zero vector is considered orthogonal to all vectors. Although in this case \(u\) and \(v\) are identical, they are not considered parallel, as only nonzero vectors can be parallel.)

25. \(\overrightarrow{QP} = (-1, -3, 2), \overrightarrow{QR} = (4, -2, -1), \) and \(\overrightarrow{QP} \cdot \overrightarrow{QR} = -4 + 6 - 2 = 0\). Thus \(\overrightarrow{QP}\) and \(\overrightarrow{QR}\) are orthogonal, so the angle of the triangle at vertex \(Q\) is a right angle.

28. Let \(u = (a, b)\) be a unit vector. By Theorem 3 we need \(u \cdot v = |u| |v| \cos 60^\circ = 3a + 4b = (1)(5)\frac{1}{2} \Rightarrow \)

\(b = \frac{5}{8} - \frac{3}{4}a\). Since \(u\) is a unit vector, \(|u| = \sqrt{a^2 + b^2} = 1 \Rightarrow a^2 + b^2 = 1 \Rightarrow a^2 + \left(\frac{5}{8} - \frac{3}{4}a\right)^2 = 1 \Rightarrow \)

\(\frac{25}{16}a^2 - \frac{15}{16}a + \frac{9}{16} = 1 \Rightarrow 100a^2 - 60a - 39 = 0\). By the quadratic formula,

\(a = \frac{-(-60) \pm \sqrt{(-60)^2 - 4(100)(-39)}}{2(100)} = \frac{60 \pm \sqrt{19200}}{200} = \frac{3 \pm 4\sqrt{3}}{10}\). If \(a = \frac{3 \pm 4\sqrt{3}}{10}\) then

\(b = \frac{5}{8} - \frac{3}{4} \left(\frac{3 + \pm 4\sqrt{3}}{10}\right) = \frac{4 - 3 \pm 4\sqrt{3}}{10}\), and if \(a = \frac{3 - 4\sqrt{3}}{10}\) then \(b = \frac{5}{8} - \frac{3}{4} \left(\frac{3 - 4\sqrt{3}}{10}\right) = \frac{4 + 3 \sqrt{3}}{10}\). Thus the two unit vectors are \(\left\{\frac{3 + 4\sqrt{3}}{10}, \frac{4 - 3 \sqrt{3}}{10}\right\} \approx (0.9928, -0.1196)\) and \(\left\{\frac{3 - 4\sqrt{3}}{10}, \frac{4 + 3 \sqrt{3}}{10}\right\} \approx (-0.3928, 0.9196)\).

43. \(|a| = \sqrt{9 + 9 + 1} = \sqrt{19}\) so the scalar projection of \(b\) onto \(a\) is \(\text{comp}_a b = \frac{a \cdot b}{|a|} = \frac{6 - 12 - \frac{1}{19}}{\sqrt{19}} = \frac{-\frac{7}{19}}{\sqrt{19}}\) while the vector projection of \(b\) onto \(a\) is \(\text{proj}_a b = -\frac{7}{\sqrt{19}} \left(\frac{3}{\sqrt{19}} \right) = -\frac{7}{19}(3i - 3j + k) = -\frac{21}{19}i + \frac{21}{19}j - \frac{7}{19}k.\)