7. $\mathbf{a} \cdot \mathbf{b} = (2\,\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = (2)(1) + (1)(-1) + (0)(1) = 1$

10. By Theorem 3, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta = (80)(50)\cos\frac{3\pi}{4} = 4000\left(-\frac{\sqrt{2}}{2}\right) = -2000\sqrt{2} \approx -2828.43$.

20. $|\mathbf{a}| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9$, $|\mathbf{b}| = \sqrt{0^2 + 4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$, and $\mathbf{a} \cdot \mathbf{b} = (8)(0) + (-1)(4) + (4)(2) = 4$.

Then $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{4}{9 \cdot 2\sqrt{5}} = \dfrac{2}{9\sqrt{5}}$ and $\theta = \cos^{-1}\left(\dfrac{2}{9\sqrt{5}}\right) \approx 84^\circ$.

23. (a) $\mathbf{a} \cdot \mathbf{b} = (9)(-2) + (3)(6) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(b) $\mathbf{a} \cdot \mathbf{b} = (4)(3) + (5)(-1) + (-2)(5) = -3 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Also, since $\mathbf{a}$ is not a scalar multiple of $\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(c) $\mathbf{a} \cdot \mathbf{b} = (-8)(6) + (12)(-9) + (4)(-3) = -168 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Because $\mathbf{a} = -\frac{4}{3}\,\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

(d) $\mathbf{a} \cdot \mathbf{b} = (3)(5) + (-1)(9) + (3)(-2) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

24. (a) $\mathbf{u} \cdot \mathbf{v} = (-5)(3) + (4)(4) + (-2)(-1) = 3 \neq 0$, so $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal. Also, $\mathbf{u}$ is not a scalar multiple of $\mathbf{v}$, so $\mathbf{u}$ and $\mathbf{v}$ are not parallel.

(b) $\mathbf{u} \cdot \mathbf{v} = (9)(-6) + (-6)(4) + (3)(-2) = -84 \neq 0$, so $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal. Because $\mathbf{u} = -\frac{3}{2}\,\mathbf{v}$, $\mathbf{u}$ and $\mathbf{v}$ are parallel.

(c) $\mathbf{u} \cdot \mathbf{v} = (c)(c) + (c)(0) + (c)(-c) = c^2 + 0 - c^2 = 0$, so $\mathbf{u}$ and $\mathbf{v}$ are orthogonal (and not parallel). (Note that if $c = 0$ then $\mathbf{u} = \mathbf{v} = \mathbf{0}$, and the zero vector is considered orthogonal to all vectors. Although in this case $\mathbf{u}$ and $\mathbf{v}$ are identical, they are not considered parallel, as only nonzero vectors can be parallel.)

25. $\overrightarrow{QP} = \langle -1, -3, 2\rangle$, $\overrightarrow{QR} = \langle 4, -2, -1\rangle$, and $\overrightarrow{QP} \cdot \overrightarrow{QR} = -4 + 6 - 2 = 0$. Thus $\overrightarrow{QP}$ and $\overrightarrow{QR}$ are orthogonal, so the angle of the triangle at vertex Q is a right angle.

28. Let $\mathbf{u} = \langle a, b\rangle$ be a unit vector. By Theorem 3 we need $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}|\,|\mathbf{v}| \cos 60^\circ \quad \Leftrightarrow \quad 3a + 4b = (1)(5)\frac{1}{2} \quad \Leftrightarrow$ $b = \frac{5}{8} - \frac{3}{4}a$. Since $\mathbf{u}$ is a unit vector, $|\mathbf{u}| = \sqrt{a^2 + b^2} = 1 \quad \Leftrightarrow \quad a^2 + b^2 = 1 \quad \Leftrightarrow \quad a^2 + \left(\frac{5}{8} - \frac{3}{4}a\right)^2 = 1 \quad \Leftrightarrow$ $\frac{25}{16}a^2 - \frac{15}{16}a + \frac{25}{64} = 1 \quad \Leftrightarrow \quad 100a^2 - 60a - 39 = 0$. By the quadratic formula,

$$a = \frac{-(-60) \pm \sqrt{(-60)^2 - 4(100)(-39)}}{2(100)} = \frac{60 \pm \sqrt{19{,}200}}{200} = \frac{3 \pm 4\sqrt{3}}{10}.$$ If $a = \dfrac{3 + 4\sqrt{3}}{10}$ then

$b = \dfrac{5}{8} - \dfrac{3}{4}\left(\dfrac{3 + 4\sqrt{3}}{10}\right) = \dfrac{4 - 3\sqrt{3}}{10}$, and if $a = \dfrac{3 - 4\sqrt{3}}{10}$ then $b = \dfrac{5}{8} - \dfrac{3}{4}\left(\dfrac{3 - 4\sqrt{3}}{10}\right) = \dfrac{4 + 3\sqrt{3}}{10}$. Thus the two unit vectors are $\left\langle \dfrac{3 + 4\sqrt{3}}{10}, \dfrac{4 - 3\sqrt{3}}{10}\right\rangle \approx \langle 0.9928, -0.1196\rangle$ and $\left\langle \dfrac{3 - 4\sqrt{3}}{10}, \dfrac{4 + 3\sqrt{3}}{10}\right\rangle \approx \langle -0.3928, 0.9196\rangle$.

43. $|\mathbf{a}| = \sqrt{9 + 9 + 1} = \sqrt{19}$ so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\operatorname{comp}_{\mathbf{a}} \mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} = \dfrac{6 - 12 - 1}{\sqrt{19}} = -\dfrac{7}{\sqrt{19}}$ while the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = -\dfrac{7}{\sqrt{19}} \dfrac{\mathbf{a}}{|\mathbf{a}|} = -\frac{7}{\sqrt{19}} \cdot \frac{1}{\sqrt{19}}(3\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}) = -\frac{7}{19}(3\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}) = -\frac{21}{19}\,\mathbf{i} + \frac{21}{19}\,\mathbf{j} - \frac{7}{19}\,\mathbf{k}$.