10. \( f(x, y) = (5y^4 \cos^2 x)/(x^4 + y^4) \). First approach \((0, 0)\) along the \(x\)-axis. Then \( f(x, 0) = 0/x^4 = 0 \) for \( x \neq 0 \), so 
\( f(x, y) \approx 0 \). Next approach \((0, 0)\) along the \(y\)-axis. For \( y \neq 0 \), \( f(0, y) = 5y^4/y^4 = 5 \), so \( f(x, y) \to 5 \). Since \( f \) has two different limits along two different lines, the limit does not exist.

11. \( f(x, y) = (y^2 \sin^2 x)/(x^4 + y^4) \). On the \(x\)-axis, \( f(x, 0) = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \) as \((x, y) \to (0, 0)\) along the \(x\)-axis. Approaching \((0, 0)\) along the line \( y = x \), 
\[ f(x, x) = \frac{x^2 \sin^2 x}{x^4 + x^4} = \frac{\sin^2 x}{2x^2} = \frac{1}{2} \left( \frac{\sin x}{x} \right)^2 \] 
\[ \lim_{x \to 0} \frac{\sin x}{x} = 1 \] 
So, \( f(x, y) \to \frac{1}{2} \). Since \( f \) has two different limits along two different lines, the limit does not exist.

12. \( f(x, y) = \frac{xy - y}{(x - 1)^2 + y^2} \). On the \(x\)-axis, \( f(x, 0) = 0/(x - 1)^2 = 0 \) for \( x \neq 1 \), so \( f(x, y) \to 0 \) as \((x, y) \to (1, 0)\) along the \(x\)-axis. Approaching \((1, 0)\) along the line \( y = x - 1 \), 
\[ f(x, x - 1) = \frac{x(x - 1) - (x - 1)}{(x - 1)^2 + (x - 1)^2} = \frac{(x - 1)^2}{2(x - 1)^2} = \frac{1}{2} \] 
So, \( f(x, y) \to \frac{1}{2} \) along this line. Thus the limit does not exist.

16. We can use the Squeeze Theorem to show that 
\[ \lim_{(x,y) \to (0,0)} \frac{xy^4}{x^4 + y^4} = 0 \]:
\[ 0 \leq \frac{|x| y^4}{x^4 + y^4} \leq |x| \text{ since } 0 \leq \frac{y^4}{x^4 + y^4} \leq 1, \text{ and } |x| \to 0 \text{ as } (x, y) \to (0, 0), \text{ so } \frac{|x| y^4}{x^4 + y^4} \to 0 \Rightarrow \frac{xy^4}{x^4 + y^4} \to 0 \text{ as } (x, y) \to (0, 0). \]

17. \[ \lim_{(x,y) \to (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2} + 1} = \lim_{(x,y) \to (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2} + 1} = \frac{\sqrt{x^2 + y^2} + 1 + 1}{\sqrt{x^2 + y^2} + 1 + 1} \]
\[ = \lim_{(x,y) \to (0,0)} \frac{(x^2 + y^2)(\sqrt{x^2 + y^2} + 1 + 1)}{x^2 + y^2} = \lim_{(x,y) \to (0,0)} (\sqrt{x^2 + y^2} + 1 + 1) = 2 \]

25. \( h(x, y) = g(f(x, y)) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6} \). Since \( f \) is a polynomial, it is continuous on \( \mathbb{R}^2 \) and \( g \) is continuous on its domain \( \{ t \mid t \geq 0 \} \). Thus \( h \) is continuous on its domain \( \{ (x, y) \mid 2x + 3y - 6 \geq 0 \} = \{ (x, y) \mid y \geq -\frac{2}{3}x + 2 \} \), which consists of all points on or above the line \( y = -\frac{2}{3}x + 2 \).

34. \( G(x, y) = \ln(1 + x - y) = g(f(x, y)) \) where \( f(x, y) = 1 + x - y \), a polynomial and hence continuous on \( \mathbb{R}^2 \), and \( g(t) = \ln t \), continuous on its domain \( \{ t \mid t > 0 \} \). Thus \( G \) is continuous on its domain \( \{ (x, y) \mid 1 + x - y > 0 \} = \{ (x, y) \mid y < x + 1 \} \), the region in \( \mathbb{R}^2 \) below the line \( y = x + 1 \).

35. \( f(x, y, z) = h(g(x, y, z)) \) where \( g(x, y, z) = x^2 + y^2 + z^2 \), a polynomial that is continuous everywhere, and \( h(t) = \arcsin t \), continuous on \( [-1, 1] \). Thus \( f \) is continuous on its domain \( \{ (x, y, z) \mid -1 \leq x^2 + y^2 + z^2 \leq 1 \} = \{ (x, y, z) \mid x^2 + y^2 + z^2 \leq 1 \} \), so \( f \) is continuous on the unit ball.

36. \( \sqrt{y - x^2} \) is continuous on its domain \( \{ (x, y) \mid y - x^2 \geq 0 \} = \{ (x, y) \mid y \geq x^2 \} \) and \( \ln z \) is continuous on its domain \( \{ z \mid z > 0 \} \), so the product \( f(x, y, z) = \sqrt{y - x^2} \ln z \) is continuous for \( y \geq x^2 \) and \( z > 0 \), that is, 
\[ \{ (x, y, z) \mid y \geq x^2, z > 0 \} \].
37. \( f(x, y) = \begin{cases} \frac{x^2 y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases} \) The first piece of \( f \) is a rational function defined everywhere except at the origin, so \( f \) is continuous on \( \mathbb{R}^2 \) except possibly at the origin. Since \( x^2 \leq 2x^2 + y^2 \), we have \( |x^2 y^3/(2x^2 + y^2)| \leq |y^3| \).

We know that \( |y^3| \to 0 \) as \( (x, y) \to (0, 0) \). So, by the Squeeze Theorem, \( \lim_{(x,y) \to (0,0)} f(x,y) = \lim_{(x,y) \to (0,0)} \frac{x^2 y^3}{2x^2 + y^2} = 0 \).

But \( f(0, 0) = 1 \), so \( f \) is discontinuous at \( (0, 0) \). Therefore, \( f \) is continuous on the set \( \{(x, y) \mid (x, y) \neq (0, 0)\} \).

40. \( \lim_{(x,y) \to (0,0)} (x^2 + y^2) \ln(x^2 + y^2) = \lim_{r \to 0^+} r^2 \ln r^2 = \lim_{r \to 0^+} \frac{\ln r^2}{1/r^2} = \lim_{r \to 0^+} \frac{(1/r^2)(2r)}{-2/r^3} \) [using l’Hospital’s Rule]

\[ = \lim_{r \to 0^+} (-r^2) = 0 \]

41. \( \lim_{(x,y) \to (0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2} = \lim_{r \to 0^+} \frac{e^{-r^2} - 1}{r^2} = \lim_{r \to 0^+} \frac{e^{-r^2}(-2r)}{2r} \) [using l’Hospital’s Rule]

\[ = \lim_{r \to 0^+} -e^{-r^2} = -e^0 = -1 \]