16. $f(x,y) = x^2y - 3y^4 \quad\Rightarrow\quad f_x(x,y) = 2x\cdot y - 0 = 2xy,\ f_y(x,y) = x^2\cdot 1 - 3\cdot 4y^3 = x^2 - 12y^3$

17. $f(x,t) = t^2e^{-x} \quad\Rightarrow\quad f_x(x,t) = t^2\cdot e^{-x}(-1) = -t^2e^{-x},\ f_t(x,t) = 2te^{-x}$

18. $f(x,t) = \sqrt{3x+4t} \quad\Rightarrow\quad f_x(x,t) = \frac{1}{2}(3x+4t)^{-1/2}(3) = \dfrac{3}{2\sqrt{3x+4t}},\ f_t(x,t) = \frac{1}{2}(3x+4t)^{-1/2}(4) = \dfrac{2}{\sqrt{3x+4t}}$

36. $u = x^{y/z} \quad\Rightarrow\quad u_x = \dfrac{y}{z}\,x^{(y/z)-1},\ u_y = x^{y/z}\ln x\cdot\dfrac{1}{z} = \dfrac{x^{y/z}}{z}\ln x,\ u_z = x^{y/z}\ln x\cdot\dfrac{-y}{z^2} = -\dfrac{yx^{y/z}}{z^2}\ln x$

42. $f(x,y) = y\sin^{-1}(xy) \quad\Rightarrow\quad f_y(x,y) = y\cdot\dfrac{1}{\sqrt{1-(xy)^2}}(x) + \sin^{-1}(xy)\cdot 1 = \dfrac{xy}{\sqrt{1-x^2y^2}} + \sin^{-1}(xy)$,

so $f_y\left(1,\frac{1}{2}\right) = \dfrac{1\cdot\frac{1}{2}}{\sqrt{1-1^2\left(\frac{1}{2}\right)^2}} + \sin^{-1}\left(1\cdot\frac{1}{2}\right) = \dfrac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} + \sin^{-1}\frac{1}{2} = \frac{1}{\sqrt{3}} + \frac{\pi}{6}$.

52. (a) $z = f(x)g(y) \quad\Rightarrow\quad \dfrac{\partial z}{\partial x} = f'(x)g(y),\quad \dfrac{\partial z}{\partial y} = f(x)g'(y)$

(b) $z = f(xy)$. Let $u = xy$. Then $\dfrac{\partial u}{\partial x} = y$ and $\dfrac{\partial u}{\partial y} = x$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du}\cdot y = yf'(u) = yf'(xy)$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du}\cdot x = xf'(u) = xf'(xy)$.

(c) $z = f\left(\dfrac{x}{y}\right)$. Let $u = \dfrac{x}{y}$. Then $\dfrac{\partial u}{\partial x} = \dfrac{1}{y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{x}{y^2}$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = f'(u)\dfrac{1}{y} = \dfrac{f'(x/y)}{y}$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = f'(u)\left(-\dfrac{x}{y^2}\right) = -\dfrac{xf'(x/y)}{y^2}$.

53. $f(x,y) = x^4y - 2x^3y^2 \quad\Rightarrow\quad f_x(x,y) = 4x^3y - 6x^2y^2,\ f_y(x,y) = x^4 - 4x^3y$. Then $f_{xx}(x,y) = 12x^2y - 12xy^2$,

$f_{xy}(x,y) = 4x^3 - 12x^2y,\ f_{yx}(x,y) = 4x^3 - 12x^2y$, and $f_{yy}(x,y) = -4x^3$.

64. $f(x,y) = \sin(2x+5y) \quad\Rightarrow\quad f_y = \cos(2x+5y)\cdot 5 = 5\cos(2x+5y),\ f_{yx} = -5\sin(2x+5y)\cdot 2 = -10\sin(2x+5y)$,

$f_{yxy} = -10\cos(2x+5y)\cdot 5 = -50\cos(2x+5y)$

71. Assuming that the third partial derivatives of f are continuous (easily verified), we can write $f_{xzy} = f_{yxz}$. Then

$f(x,y,z) = xy^2z^3 + \arcsin\left(x\sqrt{z}\right) \quad\Rightarrow\quad f_y = 2xyz^3 + 0,\ f_{yx} = 2yz^3$, and $f_{yxz} = 6yz^2 = f_{xzy}$.

72. Let $f(x,y,z) = \sqrt{1+xz}$ and $h(x,y,z) = \sqrt{1-xy}$ so that $g = f + h$. Then $f_y = 0 = f_{yx} = f_{yxz}$ and

$h_z = 0 = h_{zx} = h_{zxy}$. But (since the partial derivatives are continous on their domains) $f_{xyz} = f_{yxz}$ and $h_{xyz} = h_{zxy}$, so

$g_{xyz} = f_{xyz} + h_{xyz} = 0 + 0 = 0$.

73. By Definition 4, $f_x(3,2) = \lim\limits_{h\to 0} \dfrac{f(3+h,2) - f(3,2)}{h}$ which we can approximate by considering $h = 0.5$ and $h = -0.5$:

$f_x(3,2) \approx \dfrac{f(3.5,2) - f(3,2)}{0.5} = \dfrac{22.4 - 17.5}{0.5} = 9.8$, $f_x(3,2) \approx \dfrac{f(2.5,2) - f(3,2)}{-0.5} = \dfrac{10.2 - 17.5}{-0.5} = 14.6$. Averaging these values, we estimate $f_x(3,2)$ to be approximately 12.2. Similarly, $f_x(3,2.2) = \lim\limits_{h\to 0} \dfrac{f(3+h,2.2) - f(3,2.2)}{h}$ which we can approximate by considering $h = 0.5$ and $h = -0.5$: $f_x(3,2.2) \approx \dfrac{f(3.5,2.2) - f(3,2.2)}{0.5} = \dfrac{26.1 - 15.9}{0.5} = 20.4$,

$f_x(3,2.2) \approx \dfrac{f(2.5,2.2) - f(3,2.2)}{-0.5} = \dfrac{9.3 - 15.9}{-0.5} = 13.2$. Averaging these values, we have $f_x(3,2.2) \approx 16.8$.

To estimate $f_{xy}(3,2)$, we first need an estimate for $f_x(3,1.8)$:

$f_x(3,1.8) \approx \dfrac{f(3.5,1.8) - f(3,1.8)}{0.5} = \dfrac{20.0 - 18.1}{0.5} = 3.8$, $f_x(3,1.8) \approx \dfrac{f(2.5,1.8) - f(3,1.8)}{-0.5} = \dfrac{12.5 - 18.1}{-0.5} = 11.2$.

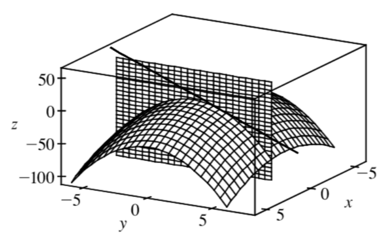
Averaging these values, we get $f_x(3,1.8) \approx 7.5$. Now $f_{xy}(x,y) = \dfrac{\partial}{\partial y}\,[f_x(x,y)]$ and $f_x(x,y)$ is itself a function of two variables, so Definition 4 says that $f_{xy}(x,y) = \dfrac{\partial}{\partial y}\,[f_x(x,y)] = \lim\limits_{h\to 0} \dfrac{f_x(x,y+h) - f_x(x,y)}{h} \quad\Rightarrow$

$f_{xy}(3,2) = \lim\limits_{h\to 0} \dfrac{f_x(3,2+h) - f_x(3,2)}{h}$. We can estimate this value using our previous work with $h = 0.2$ and $h = -0.2$:

$f_{xy}(3,2) \approx \dfrac{f_x(3,2.2) - f_x(3,2)}{0.2} = \dfrac{16.8 - 12.2}{0.2} = 23$, $f_{xy}(3,2) \approx \dfrac{f_x(3,1.8) - f_x(3,2)}{-0.2} = \dfrac{7.5 - 12.2}{-0.2} = 23.5$.

Averaging these values, we estimate $f_{xy}(3,2)$ to be approximately 23.25.

98. Setting $x = 1$, the equation of the parabola of intersection is $z = 6 - 1 - 1 - 2y^2 = 4 - 2y^2$. The slope of the tangent is $\partial z/\partial y = -4y$, so at $(1,2,-4)$ the slope is -8. Parametric equations for the line are therefore $x = 1,\ y = 2 + t$, $z = -4 - 8t$.



99. By the geometry of partial derivatives, the slope of the tangent line is $f_x(1,2)$. By implicit differentiation of $4x^2 + 2y^2 + z^2 = 16$, we get $8x + 2z\,(\partial z/\partial x) = 0 \quad\Rightarrow\quad \partial z/\partial x = -4x/z$, so when $x = 1$ and $z = 2$ we have $\partial z/\partial x = -2$. So the slope is $f_x(1,2) = -2$. Thus the tangent line is given by $z - 2 = -2(x - 1),\ y = 2$. Taking the parameter to be $t = x - 1$, we can write parametric equations for this line: $x = 1 + t,\ y = 2,\ z = 2 - 2t$.