2. $\int_0^2 \int_0^{y^2} x^2 y \, dx \, dy = \int_0^2 \left[\frac{1}{3}x^3 y\right]_{x=0}^{x=y^2} dy = \int_0^2 \frac{1}{3}y\left[(y^2)^3 - (0)^3\right] dy$

$= \int_0^2 \frac{1}{3}y^7 \, dy = \frac{1}{3}\left[\frac{1}{8}y^8\right]_0^2 = \frac{1}{3}(32 - 0) = \frac{32}{3}$

4. $\int_0^{\pi/2} \int_0^x x \sin y \, dy \, dx = \int_0^{\pi/2} [x(-\cos y)]_{y=0}^{y=x} dx = \int_0^{\pi/2} (-x\cos x + x) \, dx = \int_0^{\pi/2} (x - x\cos x) \, dx$

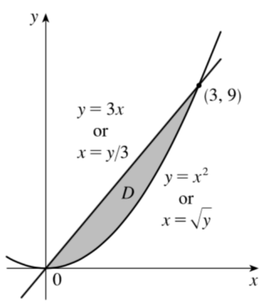
$= \left[\frac{1}{2}x^2 - (x\sin x + \cos x)\right]_0^{\pi/2}$ (by integrating by parts in the second term)

$= \left(\frac{1}{2} \cdot \frac{\pi^2}{4} - \frac{\pi}{2} - 0\right) - (0 - 0 - 1) = \frac{\pi^2}{8} - \frac{\pi}{2} + 1$

10. $\iint_D y\sqrt{x^2 - y^2} \, dA = \int_0^2 \int_0^x y\sqrt{x^2 - y^2} \, dy \, dx = \int_0^2 \left[-\frac{1}{3}(x^2 - y^2)^{3/2}\right]_{y=0}^{y=x} dx = \int_0^2 \left[0 + \frac{1}{3}(x^2)^{3/2}\right] dx$

$= \int_0^2 \frac{1}{3}x^3 \, dx = \frac{1}{3} \cdot \frac{1}{4}x^4\Big]_0^2 = \frac{1}{12}(16 - 0) = \frac{4}{3}$

14.



The curves $y = x^2$ and $y = 3x$ intersect at points $(0, 0)$, $(3, 9)$. As a type I region, D is enclosed by the lower boundary $y = x^2$ and the upper boundary $y = 3x$ for $0 \le x \le 3$, so $D = \{(x, y) \mid 0 \le x \le 3, x^2 \le y \le 3x\}$. If we describe D as a type II region, D is enclosed by the left boundary $x = y/3$ and the right boundary $x = \sqrt{y}$ for $0 \le y \le 9$, so $D = \{(x, y) \mid 0 \le y \le 9, y/3 \le x \le \sqrt{y}\}$. Thus

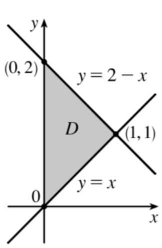
$\iint_D xy \, dA = \int_0^3 \int_{x^2}^{3x} xy \, dy \, dx = \int_0^3 \left[x \cdot \frac{1}{2}y^2\right]_{y=x^2}^{y=3x} dx = \frac{1}{2}\int_0^3 x(9x^2 - x^4) \, dx = \frac{1}{2}\int_0^3 (9x^3 - x^5) \, dx$

$= \frac{1}{2}\left[9 \cdot \frac{1}{4}x^4 - \frac{1}{6}x^6\right]_0^3 = \frac{1}{2}\left[\left(\frac{9}{4} \cdot 81 - \frac{1}{6} \cdot 729\right) - 0\right] = \frac{243}{8}$

or $\iint_D xy \, dA = \int_0^9 \int_{y/3}^{\sqrt{y}} xy \, dx \, dy = \int_0^9 \left[\frac{1}{2}x^2 y\right]_{x=y/3}^{x=\sqrt{y}} dy = \frac{1}{2}\int_0^9 \left(y - \frac{1}{9}y^2\right) y \, dy = \frac{1}{2}\int_0^9 \left(y^2 - \frac{1}{9}y^3\right) dy$

$= \frac{1}{2}\left[\frac{1}{3}y^3 - \frac{1}{9} \cdot \frac{1}{4}y^4\right]_0^9 = \frac{1}{2}\left[\left(\frac{1}{3} \cdot 729 - \frac{1}{36} \cdot 6561\right) - 0\right] = \frac{243}{8}$

28.

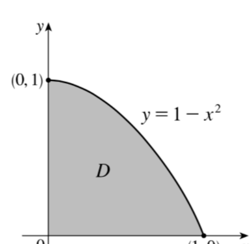


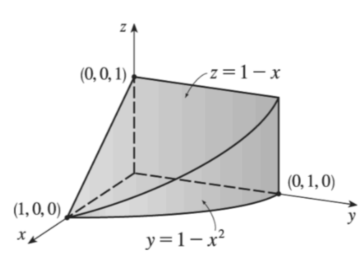
$V = \int_0^1 \int_x^{2-x} x \, dy \, dx$

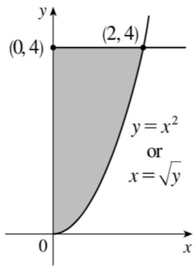
$= \int_0^1 \left[xy\right]_{y=x}^{y=2-x} dx = \int_0^1 (2x - 2x^2) \, dx$

$= \left[x^2 - \frac{2}{3}x^3\right]_0^1 = \frac{1}{3}$

40. The solid lies below the plane $z = 1 - x$ and above the region $D = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1 - x^2\}$ in the xy-plane.



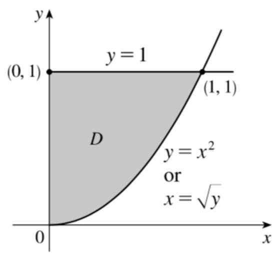


46. 

Because the region of integration is

$$D = \{(x, y) \mid x^2 \le y \le 4, 0 \le x \le 2\}$$
$$= \{(x, y) \mid 0 \le x \le \sqrt{y}, 0 \le y \le 4\}$$

we have $\int_0^2 \int_{x^2}^4 f(x, y)\, dy\, dx = \iint_D f(x, y)\, dA = \int_0^4 \int_0^{\sqrt{y}} f(x, y)\, dx\, dy$.

52. 

$$\int_0^1 \int_{x^2}^1 \sqrt{y}\, \sin y\, dy\, dx = \int_0^1 \int_0^{\sqrt{y}} \sqrt{y}\, \sin y\, dx\, dy = \int_0^1 \sqrt{y}\, \sin y\, [x]_{x=0}^{x=\sqrt{y}}\, dy$$
$$= \int_0^1 (\sqrt{y}\, \sin y)\, (\sqrt{y} - 0)\, dy = \int_0^1 y\, \sin y\, dy$$
$$= -y \cos y]_0^1 + \int_0^1 \cos y\, dy$$

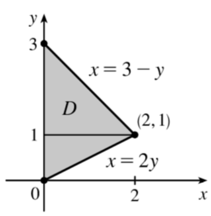
[by integrating by parts with $u = y$, $dv = \sin y\, dy$]

$$= [-y \cos y + \sin y]_0^1 = -\cos 1 + \sin 1 - 0 = \sin 1 - \cos 1$$

58. $D = \{(x, y) \mid -1 \le y \le 0,\ -1 \le x \le y - y^3\} \cup \{(x, y) \mid 0 \le y \le 1,\ \sqrt{y} - 1 \le x \le y - y^3\}$, both type II.

$$\iint_D y\, dA = \int_{-1}^0 \int_{-1}^{y-y^3} y\, dx\, dy + \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y\, dx\, dy = \int_{-1}^0 [xy]_{x=-1}^{x=y-y^3}\, dy + \int_0^1 [xy]_{x=\sqrt{y}-1}^{x=y-y^3}\, dy$$
$$= \int_{-1}^0 (y^2 - y^4 + y)\, dy + \int_0^1 (y^2 - y^4 - y^{3/2} + y)\, dy$$
$$= \left[\tfrac{1}{3}y^3 - \tfrac{1}{5}y^5 + \tfrac{1}{2}y^2\right]_{-1}^0 + \left[\tfrac{1}{3}y^3 - \tfrac{1}{5}y^5 - \tfrac{2}{5}y^{5/2} + \tfrac{1}{2}y^2\right]_0^1$$
$$= \left(0 - \tfrac{11}{30}\right) + \left(\tfrac{7}{30} - 0\right) = -\tfrac{2}{15}$$

60. T is the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$ so $A(T) = \frac{1}{2}(1)(2) = 1$. We have $0 \le \sin^4(x + y) \le 1$ for all x, y, and Property 11 gives $0 \cdot A(T) \le \iint_T \sin^4(x + y) dA \le 1 \cdot A(T) \quad \Rightarrow \quad 0 \le \iint_T \sin^4(x + y) dA \le 1$.

64. 

$$\iint_D f(x, y)\, dA = \int_0^1 \int_0^{2y} f(x, y)\, dx\, dy + \int_1^3 \int_0^{3-y} f(x, y)\, dx\, dy$$
$$= \int_0^2 \int_{x/2}^{3-x} f(x, y)\, dy\, dx$$