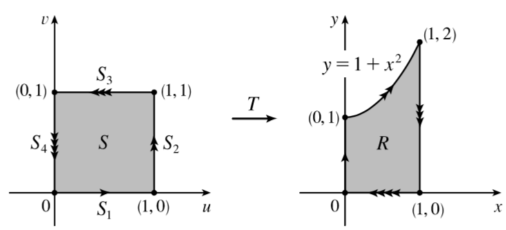
2. $x = u^2 + uv,\ y = uv^2.$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 2u+v & u \\ v^2 & 2uv \end{vmatrix} = (2u+v)(2uv) - u(v^2) = 4u^2v + 2uv^2 - uv^2 = 4u^2v + uv^2$$

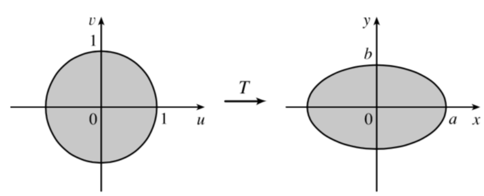
3. $x = s\cos t,\ y = s\sin t.$

$$\frac{\partial(x,y)}{\partial(s,t)} = \begin{vmatrix} \partial x/\partial s & \partial x/\partial t \\ \partial y/\partial s & \partial y/\partial t \end{vmatrix} = \begin{vmatrix} \cos t & -s\sin t \\ \sin t & s\cos t \end{vmatrix} = s\cos^2 t - (-s\sin^2 t) = s(\cos^2 t + \sin^2 t) = s$$

8. S_1 is the line segment $v = 0$, $0 \le u \le 1$, so $x = v = 0$ and $y = u(1+v^2) = u$. Since $0 \le u \le 1$, the image is the line segment $x = 0$, $0 \le y \le 1$. S_2 is the segment $u = 1$, $0 \le v \le 1$, so $x = v$ and $y = u(1+v^2) = 1 + x^2$. Thus the image is the portion of the parabola $y = 1 + x^2$ for $0 \le x \le 1$. S_3 is the segment $v = 1$, $0 \le u \le 1$, so $x = 1$ and $y = 2u$. The image is the segment $x = 1$, $0 \le y \le 2$. S_4 is described by $u = 0$, $0 \le v \le 1$, so $0 \le x = v \le 1$ and $y = u(1+v^2) = 0$. The image is the line segment $y = 0$, $0 \le x \le 1$. Thus, the image of S is the region R bounded by the parabola $y = 1 + x^2$, the x-axis, and the lines $x = 0$, $x = 1$.

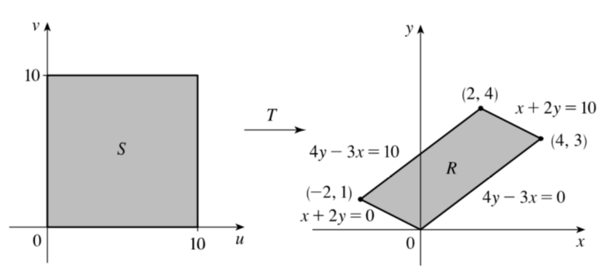


10. Substituting $u = \dfrac{x}{a}$, $v = \dfrac{y}{b}$ into $u^2 + v^2 \le 1$ gives $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \le 1$, so the image of $u^2 + v^2 \le 1$ is the elliptical region $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \le 1$.



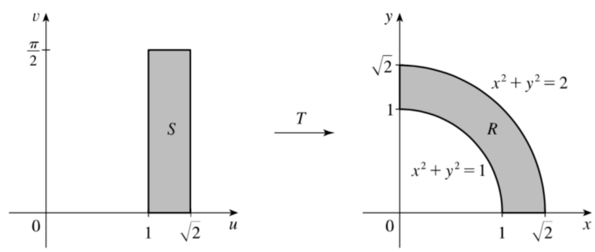
12. The boundaries of the parallelogram R are the lines $y = \frac{3}{4}x$ or $4y - 3x = 0$, $y = \frac{3}{4}x + \frac{5}{2}$ or $4y - 3x = 10$, $y = -\frac{1}{2}x$ or $x + 2y = 0$, $y = -\frac{1}{2}x + 5$ or $x + 2y = 10$. Setting $u = 4y - 3x$ and $v = x + 2y$ defines a transformation T^{-1} that maps R

in the xy-plane to the square S enclosed by the lines $u = 0$, $u = 10$, $v = 0$, $v = 10$ in the uv-plane. Solving $u = 4y - 3x$, $v = x + 2y$ for x and y gives $2v - u = 5x \quad \Rightarrow \quad x = \frac{1}{5}(2v - u)$, $u + 3v = 10y \quad \Rightarrow \quad y = \frac{1}{10}(u + 3v)$. Thus one possible transformation T is given by $x = \frac{1}{5}(2v - u)$, $y = \frac{1}{10}(u + 3v)$.



13. R is a portion of an annular region (see the figure) that is easily described in polar coordinates as

$R = \{(r, \theta) \mid 1 \le r \le \sqrt{2},\ 0 \le \theta \le \pi/2\}$. If we converted a double integral over R to polar coordinates the resulting region of integration is a rectangle (in the $r\theta$-plane), so we can create a transformation T here by letting u play the role of r and v the role of θ. Thus T is defined by $x = u\cos v$, $y = u\sin v$ and T maps the rectangle $S = \{(u, v) \mid 1 \le u \le \sqrt{2},\ 0 \le v \le \pi/2\}$ in the uv-plane to R in the xy-plane.



16. $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/4 & 1/4 \\ -3/4 & 1/4 \end{vmatrix} = \dfrac{1}{4}$, $4x + 8y = 4 \cdot \frac{1}{4}(u+v) + 8 \cdot \frac{1}{4}(v - 3u) = 3v - 5u$. R is a parallelogram bounded by the lines $x - y = -4$, $x - y = 4$, $3x + y = 0$, $3x + y = 8$. Since $u = x - y$ and $v = 3x + y$, R is the image of the rectangle enclosed by the lines $u = -4$, $u = 4$, $v = 0$, and $v = 8$. Thus

$$\iint_R (4x + 8y)\, dA = \int_{-4}^{4} \int_0^8 (3v - 5u) \left|\tfrac{1}{4}\right| dv\, du = \tfrac{1}{4} \int_{-4}^{4} \left[\tfrac{3}{2}v^2 - 5uv\right]_{v=0}^{v=8} du$$
$$= \tfrac{1}{4} \int_{-4}^{4} (96 - 40u)\, du = \tfrac{1}{4}\left[96u - 20u^2\right]_{-4}^{4} = 192$$

17. $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$, $x^2 = 4u^2$ and the planar ellipse $9x^2 + 4y^2 \le 36$ is the image of the disk $u^2 + v^2 \le 1$. Thus

$$\iint_R x^2\, dA = \iint_{u^2+v^2 \le 1} (4u^2)(6)\, du\, dv = \int_0^{2\pi} \int_0^1 (24r^2 \cos^2\theta)\, r\, dr\, d\theta = 24 \int_0^{2\pi} \cos^2\theta\, d\theta \int_0^1 r^3\, dr$$
$$= 24\left[\tfrac{1}{2}x + \tfrac{1}{4}\sin 2x\right]_0^{2\pi} \left[\tfrac{1}{4}r^4\right]_0^1 = 24(\pi)\left(\tfrac{1}{4}\right) = 6\pi$$

23. Letting $u = x - 2y$ and $v = 3x - y$, we have $x = \frac{1}{5}(2v - u)$ and $y = \frac{1}{5}(v - 3u)$. Then $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -1/5 & 2/5 \\ -3/5 & 1/5 \end{vmatrix} = \dfrac{1}{5}$ and R is the image of the rectangle enclosed by the lines $u = 0$, $u = 4$, $v = 1$, and $v = 8$. Thus

$$\iint_R \frac{x - 2y}{3x - y}\, dA = \int_0^4 \int_1^8 \frac{u}{v} \left|\frac{1}{5}\right| dv\, du = \frac{1}{5} \int_0^4 u\, du \int_1^8 \frac{1}{v}\, dv = \tfrac{1}{5}\left[\tfrac{1}{2}u^2\right]_0^4 \left[\ln|v|\right]_1^8 = \tfrac{8}{5}\ln 8.$$

26. Letting $u = 3x$, $v = 2y$, we have $9x^2 + 4y^2 = u^2 + v^2$, $x = \frac{1}{3}u$, and $y = \frac{1}{2}v$. Then $\dfrac{\partial(x,y)}{\partial(u,v)} = \dfrac{1}{6}$ and R is the image of the quarter-disk D given by $u^2 + v^2 \le 1$, $u \ge 0$, $v \ge 0$. Thus

$$\iint_R \sin(9x^2 + 4y^2)\, dA = \iint_D \tfrac{1}{6}\sin(u^2 + v^2)\, du\, dv = \int_0^{\pi/2} \int_0^1 \tfrac{1}{6}\sin(r^2)\, r\, dr\, d\theta = \tfrac{\pi}{12}\left[-\tfrac{1}{2}\cos r^2\right]_0^1 = \tfrac{\pi}{24}(1 - \cos 1)$$