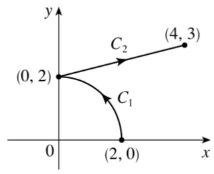
3. Parametric equations for C are $x = 4\cos t,\ \ y = 4\sin t,\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Then

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} (4\cos t)(4\sin t)^4 \sqrt{(-4\sin t)^2 + (4\cos t)^2}\,dt = \int_{-\pi/2}^{\pi/2} 4^5 \cos t\,\sin^4 t\,\sqrt{16(\sin^2 t + \cos^2 t)}\,dt$$
$$= 4^5 \int_{-\pi/2}^{\pi/2} (\sin^4 t\cos t)(4)\,dt = (4)^6\left[\tfrac{1}{5}\sin^5 t\right]_{-\pi/2}^{\pi/2} = 4^6 \cdot \tfrac{2}{5} = 1638.4$$

8.



$C = C_1 + C_2$

On C_1: $x = 2\cos t \ \Rightarrow\ dx = -2\sin t\,dt,\ \ y = 2\sin t \ \Rightarrow$
$dy = 2\cos t\ dt,\ \ 0 \le t \le \frac{\pi}{2}$.

On C_2: $x = 4t \ \Rightarrow\ dx = 4\,dt,\ y = 2 + t \ \Rightarrow$
$dy = dt,\ \ 0 \le t \le 1$.

Then

$$\int_C x^2\,dx + y^2\,dy = \int_{C_1} x^2\,dx + y^2\,dy + \int_{C_2} x^2\,dx + y^2\,dy$$
$$= \int_0^{\pi/2} (2\cos t)^2(-2\sin t\,dt) + (2\sin t)^2(2\cos t\,dt) + \int_0^1 (4t)^2(4\,dt) + (2+t)^2\,dt$$
$$= 8\int_0^{\pi/2}(-\cos^2 t\sin t + \sin^2 t\cos t)\,dt + \int_0^1 (65t^2 + 4t + 4)\,dt$$
$$= 8\left[\tfrac{1}{3}\cos^3 t + \tfrac{1}{3}\sin^3 t\right]_0^{\pi/2} + \left[\tfrac{65}{3}t^3 + 2t^2 + 4t\right]_0^1 = 8\left(\tfrac{1}{3} - \tfrac{1}{3}\right) + \tfrac{65}{3} + 2 + 4 = \tfrac{83}{3}$$

9. $x = \cos t,\ \ y = \sin t,\ \ z = t,\ \ 0 \le t \le \pi/2$. Then by Formula 9,

$$\int_C x^2y\,ds = \int_0^{\pi/2} (\cos t)^2(\sin t)\sqrt{\left(\tfrac{dx}{dt}\right)^2 + \left(\tfrac{dy}{dt}\right)^2 + \left(\tfrac{dz}{dt}\right)^2}\,dt$$
$$= \int_0^{\pi/2} \cos^2 t\sin t\,\sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}\,dt = \int_0^{\pi/2}\cos^2 t\sin t\,\sqrt{\sin^2 t + \cos^2 t + 1}\,dt$$
$$= \sqrt{2}\int_0^{\pi/2}\cos^2 t\sin t\,dt = \sqrt{2}\left[-\tfrac{1}{3}\cos^3 t\right]_0^{\pi/2} = \sqrt{2}\left(0 + \tfrac{1}{3}\right) = \tfrac{\sqrt{2}}{3}$$

17. (a) Along the line $x = -3$, the vectors of $\mathbf{F}$ have positive y-components, so since the path goes upward, the integrand $\mathbf{F}\cdot\mathbf{T}$ is always positive. Therefore $\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = \int_{C_1}\mathbf{F}\cdot\mathbf{T}\,ds$ is positive.

(b) All of the (nonzero) field vectors along the circle with radius 3 are pointed in the clockwise direction, that is, opposite the direction to the path. So $\mathbf{F}\cdot\mathbf{T}$ is negative, and therefore $\int_{C_2}\mathbf{F}\cdot d\mathbf{r} = \int_{C_2}\mathbf{F}\cdot\mathbf{T}\,ds$ is negative.

18. Vectors starting on C_1 point in roughly the same direction as C_1, so the tangential component $\mathbf{F}\cdot\mathbf{T}$ is positive. Then $\int_{C_1}\mathbf{F}\cdot d\mathbf{r} = \int_{C_1}\mathbf{F}\cdot\mathbf{T}\,ds$ is positive. On the other hand, no vectors starting on C_2 point in the same direction as C_2, while some vectors point in roughly the opposite direction, so we would expect $\int_{C_2}\mathbf{F}\cdot d\mathbf{r} = \int_{C_2}\mathbf{F}\cdot\mathbf{T}\,ds$ to be negative.

21. $\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 \left\langle \sin t^3, \cos(-t^2), t^4\right\rangle \cdot \left\langle 3t^2, -2t, 1\right\rangle dt$

$$= \int_0^1 (3t^2\sin t^3 - 2t\cos t^2 + t^4)\,dt = \left[-\cos t^3 - \sin t^2 + \tfrac{1}{5}t^5\right]_0^1 = \tfrac{6}{5} - \cos 1 - \sin 1$$

39. $W = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi}\langle t - \sin t, 3 - \cos t\rangle \cdot \langle 1 - \cos t, \sin t\rangle\,dt$

$$= \int_0^{2\pi}(t - t\cos t - \sin t + \sin t\cos t + 3\sin t - \sin t\cos t)\,dt$$
$$= \int_0^{2\pi}(t - t\cos t + 2\sin t)\,dt = \left[\tfrac{1}{2}t^2 - (t\sin t + \cos t) - 2\cos t\right]_0^{2\pi}$$

[integrate by parts in the second term]

$$= 2\pi^2$$