Complex Analysis, Fall 2017

Problem Set 2

Due: September 19 in class

1. If R_1 is the radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$ and R_2 is the radius of convergence of $\sum_{n=0}^{\infty} b_n z^n$, then show that the radius of convergence of $\sum_{n=0}^{\infty} a_n b_n z^n$ is at least $R_1 R_2$.

2. For what value of z is

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} (\frac{z}{1+z})^n$$

convergent?

3. Determine all values of i^i and $(-1)^{2i}$.

4. Assume $\{a_n\}$ is a non-increasing sequence of real numbers, and $\lim_{n\to\infty} a_n = 0$. For any $\delta > 0$, show that $\sum_{n=0}^{\infty} a_n z^n$ is uniformly convergent on $|z-1| \geq \delta, |z| \leq 1$.

5. Describe the image of the right half plane (Re z > 0) under the function $z \mapsto$ Log z.

6. Prove $\cos(z+w) = \cos z \cos w - \sin z \sin w$ for any two complex numbers w and z.

7. Let $\{a_n\}$ be the Fibonacci sequence: $a_0 = a_1 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$, and let α_1 and α_2 be the 2 roots of the polynomial $x^2 - x - 1$.

a) Use induction to show that there are numbers A and B such that

$$a_n = A\alpha_1^n + B\alpha_2^n$$

for every $n \geq 0$.

(b) What is the radius of convergence R of the power series $\sum_{n=0}^{\infty} a_n z^n$?

(c) What does the power series converge to in $\{|z| < R\}$? (It is a general fact that a power series converges to a quotient of polynomials if and only if its coefficients satisfy a recurrence relation.)