Algebra I, Fall 2016

Problem Set 6

Due: October 20 in class

1. (Lang, II.3) Let A be a PID and S a multiplicative subset not containing 0. Show $S^{-1}A$ is a PID.

2. (Lang, II.4) Let A be a UFD and S a multiplicative subset not containing 0. Show $S^{-1}A$ is a UFD and that the prime elements of $S^{-1}A$ are of the form up where u is a unit in $S^{-1}A$ and p is a prime element in A such that $(p) \cap S = \emptyset$.

3. (Lang, II.7) Suppose R is a PID and $a_1, \ldots, a_n$ are non-zero elements of R. Let $(a_1, \ldots, a_n) = (d)$. Show d is the greatest common divisor of the a_i. (so it divides each a_i, and if c divides each a_i, then c divides d.)

4. Show the subring $\mathbf{Z}[2i] = \{a + 2bi \mid a, b \in \mathbf{Z}\}$ of the Guassian integers is not a UFD by showing $4 = 2 \cdot 2 = (-2i) \cdot (2i)$ gives two factorization of 4 into product of irreducible elements.

5. Let $R = \mathbf{Z}[i]$ and $d(a + bi) = a^2 + b^2$. Let $\alpha = 11 + 3i$ and $\beta = 1 + 8i$.

(1) Write $\alpha = \beta q + r$ in R with $d(r) < d(\beta)$ using the method we discussed in class.

(2) Find the gcd of α and β by showing $\gcd(\alpha, \beta) = \gcd(\beta, r)$, dividing β by r, and continuing the process until the remainder is zero. (this is the Euclidean algorithm)