To show part (c) implies part (a) in Question 2 you can do the following: Let $\alpha : M' \to M$ be an injective R-homomorphism and $f : M' \to Q$ a R-homomorphism. Define a R-homomorphism $\beta : M' \to Q \oplus M$ by $\beta(x) = (-f(x), \alpha(x))$. The image of β is a submodule of $Q \oplus M$. Let $N = (Q \oplus M)/Im(\beta)$. There are two R-homomorphisms $\tilde{\alpha} : Q \to N$, sending q to $(q, 0) + Im(\beta)$ and $\tilde{f} : M \to N$ sending m to $(0, m) + Im(\beta)$.

$$\begin{array}{ccc} M' & \xrightarrow{\alpha} & M \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle \tilde{f}} \\ Q & \xrightarrow{\tilde{\alpha}} & N \end{array}$$

Show that

- $\tilde{f} \circ \alpha = \tilde{\alpha} \circ f$
- $\tilde{\alpha}$ is injective
- by part (c) there is a r-homomorphism $\gamma : N \to Q$ such that $q \circ \tilde{\alpha} = id_Q$
- $\gamma \circ \tilde{f} : M \to Q$ is the extension of f to M. (i.e. $f = (\gamma \circ \tilde{f}) \circ \alpha$)