1.(1 pt) Now for some practice working with complex numbers:

Calculate
$(4+7i)+(3-i) =$ ______
$(4+7i)-(3-i) =$ ______
$(4+7i)(3-i) =$ ______
The complex conjugate of (1 +i) is (1-i). In general to obtain the complex conjugate reverse the sign of the imaginary part. (Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the x axis. The complex conjugate of a complex number z is written with a bar over it:

$$\overline{z}$$

and read as "z bar".

Notice that if $z = a + ib$ then

$$(z)(\overline{z}) = |z|^2 = a^2 + b^2$$

which is also the square of the distance of the point z from the origin. (Plot z as a point in the "complex" plane in order to see this.)

If $z = 4+7i$ then $\overline{z} =$ ______ and $|z| =$ ______

You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.

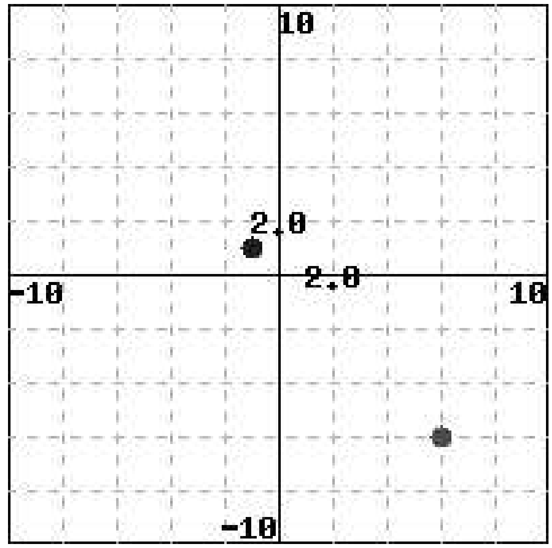
$$\frac{4+7i}{3-i} = \_\_ \quad + i\_\_$$

Two convenient functions to know about pick out the real and imaginary parts of a complex number.

$\Re(a+ib) = a$ (the real part (coordinate) of the complex number) and
$\Im(a+ib) = b$ (the imaginary part (coordinate) of the complex number. Re and Im are linear functions – now that you know about linear behavior you may start noticing it often.

2.(1 pt) More on complex numbers. (For additional help check out the appendix in Stewart's Calculus book, from MTH161, MTH162. There is an entire appendix of hints for working with complex numbers.)

An apology: The exponents don't print very well on the screen version of this problem. You can get a better idea of what the notation looks like from the hard copy and/or you can use the "typeset" mode to get a better printing. Unfortunately in typset mode you won't be able to enter the answers which are within equations.



The red point represents the complex number $z_1 =$ ___ and the blue point represents the complex number $z_2 =$ ______
$|z_1| =$ ______

We can also write these complex numbers in polar coordinates (r,θ). The angle is sometimes called the "argument" of the complex number and r is called the "modulus" or the absolute value of the number.

By comparing Taylor series we find that

$$e^{i\theta} = \cos(\theta) + i\sin(\theta).$$

This is a very important and very useful formula. One use is to relate the polar coordinate and cartesian coordinate formulas for the complex number. If z can be represented by both coordinates $x+iy$ and by polar coordinates r,θ then

$$re^{i\theta} = r\cos(\theta) + ir\sin(\theta) = x + iy = z$$

Represent -1+i (the blue point) in polar coordinates (use an angle between -pi and pi):

$$\_\_ \quad e^{i\_\_}$$

Represent 6-6i (the red point) in polar coordinates:

$$\_\_ \quad e^{i\_\_}$$

Using the law of exponents it is really easy to multiply complex numbers represented in polar coordinates – the angles just add!

$$(r_1 e^{i\theta_1} r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

Find (6-6i)*(-1+i) using polar coordinates and your answer above:

$$(6-6i)(-1+i) = \_\_ \quad e^{i\_\_}$$

Check your answer by doing the standard multiplication and then converting to polar coordinates. Can you plot this number on the graph?

3.(1 pt)

Enter T or F depending on whether the statement is true or false. (You must enter T or F – True and False will not work.)

__ 1. If a Laurent series converges in an annulus $r < |z| < R$ then differentiating the Laurent series term by term produces a new Laurent series which converges on an annulus $\hat{r} < |\hat{z}| < \hat{R}$ but the new annulus might be smaller than the original annulus. (i.e $r < \hat{r}$ and $\hat{R} < R$).

__ 2. A function $f(z)$, analytic in a domain D which contains the annulus $1 < |z| < 4$can always be expanded in a convergent power series (with no negative exponents) centered at $2i$ and converging in the disk $|z - 2i| < 1$.

__ 3. A function $f(z)$ which is analytic on the annulus $r < |z| < R$ can be represented by a Laurent series centered at 0, which converges for all z in the annulus.

__ 4. $f(z) = z^{-\frac{1}{2}}$ can be expanded as a Laurent series convergent in the annulus $0 < |z| < \infty$. It has a simple pole at $z = 0$.

__ 5. A function $f(z)$ which is analytic on a domain D can always be expanded in a power series which converges on the smallest disk containing the domain D.

4.(1 pt)

Enter T or F depending on whether the statement is true or false.

(You must enter T or F – True and False will not work.)

__ 1. If $f(z)$ is analytic in a domain D and D is simply connected, then $\int_\Gamma f(z)\,dz$ is zero for every closed loop Γ lying within D.

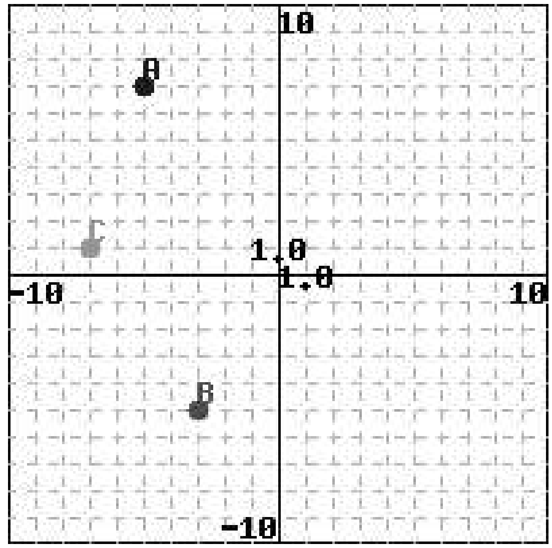
__ 2. If f is analytic in a domain D then $\int_\Gamma f(z)\,dz = 0$ for any closed loop lying in D.

__ 3. If $f(z) = \frac{d}{dz}g(z)$ on a domain D ($g(z)$ is analytic in D) then the integral $\int_\Gamma f(z)\,dz$ is zero for every closed path Γ lying within D.

__ 4. If $\int_\Gamma f(z)\,dz$ is zero for every closed path Γ lying within D then $f(z) = \frac{d}{dz}g(z)$ for some anlalytic function $g(z)$ and defined on all of D.

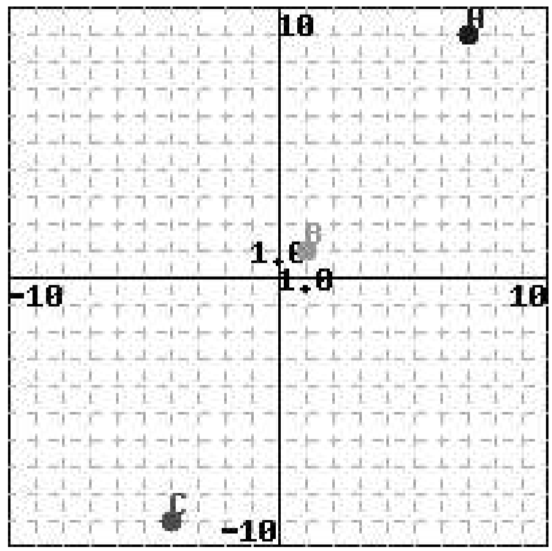
__ 5. If $f(z)$is analytic in a domain D(D is not necessarily simply connected) and the loop Γ_1 can be deformed to the loop Γ_2 within D, then $Re(\int_{\Gamma_1} f(z)\,dz) = Re(\int_{\Gamma_2} f(z)\,dz)$

5.(1 pt) **Enter the complex coordinates of the following points:**



A: ___ $+\,i$ ___ B: ___ $+\,i$ ___ C: ___ $+\,i$ ___

6.(1 pt) **Enter the complex coordinates of the following points:**



A: _____ B: _____ C: _____

7.(1 pt)

Write the following in $a + bi$ form:

(a) $-3(\frac{i}{2})$ = _____ $+\,i$ _____

(b) $(-4+i) - (3+3i)$ = _____ $+\,i$ _____

(c) $\frac{1}{i}$ = _____ $+\,i$ _____

8.(1 pt)

Write the following in $a + bi$ form:

(a) $(-1+i)^2$ = _____ $+\,i$ _____

(b) $\frac{-2+5i}{\frac{i}{5}}$ = _____ $+\,i$ _____

(c) $\frac{-2+5i}{\frac{1}{5}}$ = _____ $+\,i$ _____

9.(1 pt)

Write the following in $a + bi$ form:

(a) $(2+i)^2$ = _____ $+\,i$ _____

(b) $i(\pi - 2i)$ = _____ $+\,i$ _____

(c) $\frac{-4+i}{i}$ = ________ $+ i$ ________

10.(1 pt)
Write the following in $a + bi$ form:
Answer as a rational.
(a) $\frac{-4+i}{-2+2i}$ = ________ $+ i$ ________
(b) $\frac{-2}{2i} + \frac{-5}{5i}$ = ________ $+ i$ ________
(c) $(-i)^3$ = ________ $+ i$ ________

11.(1 pt)
Write the following in $a + bi$ form:
(a) $(\frac{2+i}{i-(1-2i)})^2$ = ________ $+ i$ ________
(b) $(i)^2(4+i)^2$ = ________ $+ i$ ________

12.(1 pt)
Write the following in $a + bi$ form:
(a) $(-4+2i)(1-5i)(-2+4i)$ = ________ $+ i$ ________
(b) $((-3+i)^2-5)i$ = ________ $+ i$ ________

13.(1 pt)
Calculate the following:
(a) i^2 = ________
(b) i^3 = ________
(c) i^4 = ________
(d) i^5 = ________
(e) i^{51} = ________
(b) i^0 = ________
(b) i^{-1} = ________
(b) i^{-2} = ________
(f) i^{-3} = ________
(b) i^{-23} = ________

14.(1 pt)
Let $z = -3+5i$
Calculate the following:
(a) $z^2 + 2z+1$ = ________ $+ i$ ________
(a) $z^2 + iz - (-4 + i)$ = ________ $+ i$ ________
(a) $\frac{(z-3)^2}{z+i}$ = ________ $+ i$ ________

15.(1 pt)
Solve the following for z:
(a) $iz = 4 - zi$
z = ________ $+ i$ ________
(b) $\frac{z}{1-z} = 1 - 5i$
z = ________ $+ i$ ________
(c) $(2 - i)z + 8z^2 = 0$
(This question has two solutions, one of which is 0, find the other)
z = ________ $+ i$ ________
(d) $z^2 + 16 = 0$
z = ________ $+ i$ ________

16.(1 pt)
Calculate:
(a) $\left|\frac{2+i}{-4-i}\right|$ = ________
(b) $\left|\overline{(1 + i)(3-4i)(4-i)}\right|$ = ________
(c) $\left|\frac{i(4+i)^3}{(2-3i)^2}\right|$ = ________
(d) $\left|\frac{(\pi+1)^{100}}{(\pi-1)^{100}}\right|$ = ________

17.(1 pt) Answer the following questions (T or F):
__ 1. $Arg\,\overline{z} = -Arg\,z$, if z is not real.
__ 2. $arg\,z = Arg\,z + 2\pi k, (k = 0, \pm1, \pm2, \pm3...)$ and if $z \neq 0$.
__ 3. $Arg\,\frac{z_1}{z_2} = Arg\,z_1 - Arg\,z_2$, if $z_1 \neq 0$, $z_2 \neq 0$.
__ 4. $Arg(0)$ is undefined.
__ 5. $Arg\,z_1z_2 = Arg\,z_1 + Arg\,z_2$, if $z_1 \neq 0$, $z_2 \neq 0$.

18.(1 pt)
Place the following in order:
(a) $|z_2| - |z_1|$, (b) $|z_2| - |z_1|$, (c) $|z_1| + |z_2|$, (d) $|z_1 + z_2|$
________ $\leq$ ________ $\leq$ ________ $\leq$ ________

19.(1 pt)
Calculate, answers are in the form re^{θ} :
(a) $-\frac{a}{b}$
r = ________ θ = ________
(b) $a + ai$
r = ________ θ = ________
(c) $b + bi$
r = ________ θ = ________
(d) $c + ci$
r = ________ θ = ________

20.(1 pt)
Calculate, answers are in the form re^{θ} :
(a) $-\pi i$
r = ________ θ = ________
(b) $-2\sqrt{3} - 4i$
r = ________ θ = ________
(c) $(1 - i)(-\sqrt{3} + i)$
r = ________ θ = ________
(d) $(\sqrt{2} - 4i)^2$
r = ________ θ = ________
(e) $\frac{-3+\sqrt{2}i}{3+2i}$
r = ________ θ = ________
(f) $\frac{-\sqrt{7}(1+i)}{\sqrt{3}+i}$
r = ________ θ = ________

21.(1 pt) Write each of the given numbers in the form $a+bi$:
(a) $e^{-\frac{i\pi}{4}}$
________ + ________ i
(b) $\frac{e^{(1+i3\pi)}}{e^{(-1+\frac{i\pi}{2})}}$
________ + ________ i
(c) e^{e^i}
________ + ________ i

22.(1 pt) Write each of the given numbers in the form $a+bi$:
(a) $\frac{e^{3i}-e^{-3i}}{2i}$
________ + ________ i
(b) $2e^{(3+\frac{i\pi}{6})}$
________ + ________ i
(c) $e^{(4e^{(\frac{i\pi}{3})})}$
________ + ________ i

23.(1 pt) Write each of the given numbers in the polar form re^{θ} :

(a) $\frac{7-i}{3}$

$r =$ ______ $\theta =$ ______

(b) $-8\pi(3+i\sqrt{2})$

$r =$ ______ $\theta =$ ______

(c) $(1+i)^6$

$r =$ ______ $\theta =$ ______

24.(1 pt) Write each of the given numbers in the polar form re^{θ} :

(a) $(cos\frac{2\pi}{9}+isin\frac{2\pi}{9})^3$

$r =$ ______ $\theta =$ ______

(b) $\frac{2+2i}{-\sqrt{3}+i}$

$r =$ ______ $\theta =$ ______

(c) $\frac{2i}{3e^{(4+i)}}$

$r =$ ______ $\theta =$ ______

25.(1 pt) Determine which of the following properties of the real exponential function remain true for the complex exponential (i.e., for x replaced by z).

Answer T or F:

__ 1. $e^x$is a one-to-one function.
__ 2. e^x is never zero.
__ 3. $e^{-x} = \frac{1}{e^x}$.
__ 4. e^x is defined for all x.

26.(1 pt) Which of the following sets are open?

- A. $0 < |z-2| < 3$
- B. $|z-1+i| \leq 3$
- C. $|z| \geq 2$
- D. $-1 < \ Im\, z \leq 1$
- E. $(Re\ z)^2 > 1$
- F. $|Arg\ z| < \frac{\pi}{4}$

27.(1 pt) Which of the given sets are bounded?

- A. $-1 < \ Im\, z \leq 1$
- B. $|z| \geq 2$
- C. $(Re\ z)^2 > 1$
- D. $|Arg\ z| < \frac{\pi}{4}$
- E. $|z-1+i| \leq 3$
- F. $0 < |z-2| < 3$

28.(1 pt) Which of the given sets are regions?

- A. $|z| \geq 2$
- B. $0 < |z-2| < 3$
- C. $(Re\ z)^2 > 1$
- D. $|Arg\ z| < \frac{\pi}{4}$
- E. $-1 < \ Im\, z \leq 1$
- F. $|z-1+i| \leq 3$

29.(1 pt) Which of the given sets are closed regions?

- A. $|z-1+i| \leq 3$
- B. $|z| \geq 2$
- C. $|Arg\ z| < \frac{\pi}{4}$
- D. $-1 < \ Im\, z \leq 1$
- E. $0 < |z-2| < 3$
- F. $(Re\ z)^2 > 1$

30.(1 pt) Which of the given sets are domains?

- A. $0 < |z-2| < 3$
- B. $(Re\ z)^2 > 1$
- C. $|Arg\ z| < \frac{\pi}{4}$
- D. $-1 < \ Im\, z \leq 1$
- E. $|z| \geq 2$
- F. $|z-1+i| \leq 3$

31.(1 pt) Find all the values of the following :

(1) $(-81)^{\frac{1}{4}}$

Place all answers in the following blank, seperated by commas:

______

(2) $1^{\frac{1}{5}}$

Place all answers in the following blank, seperated by commas:

______

(3) $i^{\frac{1}{4}}$

Place all answers in the following blank, seperated by commas:

______

32.(1 pt) Find all the values of the following :

(1) $(1-i\sqrt{3})^{\frac{1}{3}}$

Place all answers in the following blank, seperated by commas:

______

(2) $(i-1)^{\frac{1}{2}}$

Place all answers in the following blank, seperated by commas:

______

(3) $(\frac{2i}{1+i})^{\frac{1}{6}}$

Place all answers in the following blank, seperated by commas:

______

33.(1 pt) Solve the following equations for z, find all solutions :

(1) $2z^2 + z + 3 = 0$

Place all answers in the following blank, seperated by commas:

______

(2) $z^2 - (3-2i)z + 1 - 3i = 0$

Place all answers in the following blank, seperated by commas:

______

(3) $z^2 - 2z + i = 0$

Place all answers in the following blank, seperated by commas:

______