1.(1 pt) Write each of the following functions in the form $w = u(x,y) + iv(x,y)$:

(1) $f(z) = 3z^2 + 2z + 4i + 1$

______ $+ i$ ______

(2) $h(z) = \frac{5z + 5i}{2z^2 + 4}$

______ $+ i$ ______

(3) $F(z) = e^{4z}$

______ $+ i$ ______

2.(1 pt) Write each of the following functions in the form $w = u(x,y) + iv(x,y)$:

(1) $g(z) = \frac{2}{z}$

______ $+ i$ ______

(2) $q(z) = \frac{2z^2 + 3}{|z - 4|}$

______ $+ i$ ______

(3) $G(z) = e^z + e^{-z}$

______ $+ i$ ______

3.(1 pt) Decide whether each of the following sequences converges, and if so, type the limit in the answer blank.

If it does not converge, type "*DNC*" :

(1) $z_n = \frac{i}{n}$

______

(2) $z_n = i(-1)^n$

______

(3) $z_n = Arg(-1 + \frac{i}{n})$

______

(4) $z_n = \frac{n(5 + 4i)}{n + 1}$

______

(5) $z_n = (\frac{1 - i}{4})^n$

______

(6) $z_n = exp(\frac{2n\pi i}{5})$

______

4.(1 pt) A uniformly charged infinite rod, standing perpendicular to the z-plane at the point z_0, generates an electric field at every point in the plane. The intensity of this field varies inversely as the distance from z_0 to the point and is directed along the line from z_0 to the point.

If three such rods are located at the points $5 + 3i$, $-5 + 3i$, and 0, find the positions of equilibrium (i.e., the points where the vector sum of the fields is zero).(Hint: $F(z) = \frac{1}{(\bar{z} - \bar{z}_0\ )}$)

Enter all answers in the following answer blank, seperated by commas :

______

5.(1 pt) Find each of the following limits:

(1) $\lim_{z\to 3+4i} (z - 7i)^2 =$ ______

(2) $\lim_{z\to 3i} \frac{z^2 + 9}{z - 3i} =$ ______

(3) $\lim_{\Delta z\to 0} \frac{(z_0 + \Delta z)^2 - z_0^2}{\Delta z} =$ ______ (Hint: You may use z_0 in your answer, simply write it as z.)

6.(1 pt) Find each of the following limits:

(1) $\lim_{z\to 5} \frac{z^2 + 4}{iz} =$ ______

(2) $\lim_{z\to i} \frac{z^2 + 1}{z^4 - 1} =$ ______

(3) $\lim_{z\to 1+2i} |z^2 - 1| =$ ______

7.(1 pt) Find each of the following limits:

(1) $\lim_{z\to 0} e^z =$ ______

(2) $\lim_{z\to 2\pi i} e^z - e^{-z} =$ ______

(3) $\lim_{z\to \frac{\pi i}{2}} (z + 4)e^z =$ ______

(4) $\lim_{z\to \pi i} exp(\frac{z^2 + \pi^2}{z + \pi i}) =$ ______

8.(1 pt) Find the derivatives of the following functions with respect to z:

(1) $f(z) = 3z^3 + 4z^2 + iz + 10$

$f'(z) =$ ______

(2) $f(z) = (z^2 - 3i)^{-8}$

$f'(z) =$ ______

(3) $f(z) = \frac{z^2 - 9}{iz^3 + 2z + \pi}$

$f'(z) =$ ______

9.(1 pt) Find the derivatives of the following functions:

(1) $f(z) = \frac{(z + 5)^5}{(z^2 + 5iz + 6)^8}$ ______

(2) $f(z) = 8i(z^3 - 5)^4(z^2 + 5iz)^{100}$ ______

10.(1 pt) For each of the following determine the points at which the function is not analytic:

Enter all answers in the answer blank seperated by commas.

(1) $\frac{1}{z - 1 + 2i}$ ______

(2) $\frac{iz^3 + 2z}{z^2 + 1}$ ______

(3) $\frac{3z - 1}{z^2 + z + 6}$ ______

(4) $z^2(2z^2 - 3z + 1)^{-2}$ ______

11.(1 pt)

Use L'Hospital's rule to find $\lim_{z\to i} \frac{1 + z^2}{1 + z^{18}}$:

______

12.(1 pt) Let $f(z)$ and $g(z)$ be entire functions. Decide which of the following statements are always true.

Answer T or F:

__ 1. $5f(z) + ig(z)$ is entire.
__ 2. $f(g(z))$ is entire.
__ 3. $f(z)^3$ is entire.
__ 4. $f(z)/g(z)$ is entire.
__ 5. $f(1/z)$ is entire.
__ 6. $g(z^2 + 2)$ is entire.
__ 7. $f(z)g(z)$ is entire.

Find the harmonic conjugate of each harmonic function u.
(use a as your constant of integration.)

(1) $u = 5y$

______

(2) $u = 7e^x sin(y)$

______

(3) $u = 3xy - 4x + 4y$

______

Find the harmonic conjugate of each harmonic function u.

(use a as your constant of integration.)
(1) $u = sin(x)cosh(y)$

---

(2) $u = ln|z|$ for $Re z > 0$

---

(3) $u = Im(e^{z^2})$

Find a function $\phi(x,y)$ that is harmonic in the infinite vertical strip $\{z : -1 \leq Re\ z \leq 3\}$ and takes the value 1 on the left edge and the value 5 on the right edge.

Find a function $\phi(x,y)$ that is harmonic in the region of the first quadrant between the curves $xy = 2$ and $xy = 4$ and takes on the values 5 on the lower edge and the value 7 on the upper edge.[Hint: Begin by considering z^2.]
