

1.(1 pt) Find the point on the line  $-5x + 4y - 4 = 0$  which is closest to the point  $(-3, -3)$ .  
 ( \_\_\_\_\_ , \_\_\_\_\_ )

2.(1 pt) A rectangle is inscribed with its base on the  $x$ -axis and its upper corners on the parabola  $y = 1 - x^2$ . What are the dimensions of such a rectangle with the greatest possible area?  
 Width = \_\_\_\_\_ Height = \_\_\_\_\_

3.(1 pt) A cylinder is inscribed in a right circular cone of height 3.5 and radius (at the base) equal to 4. What are the dimensions of such a cylinder which has maximum volume?  
 Radius = \_\_\_\_\_ Height = \_\_\_\_\_

4.(1 pt) If 2200 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.  
 Volume = \_\_\_\_\_ cubic centimeters.

5.(1 pt) A fence 6 feet tall runs parallel to a tall building at a distance of 5 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building? \_\_\_\_\_

6.(1 pt) A fence 3 feet tall runs parallel to a tall building at a distance of 2 feet from the building. We want to find the length of the shortest ladder that will reach from the ground over the fence to the wall of the building.

Here are some hints for finding a solution:

Use the angle that the ladder makes with the ground to define the position of the ladder and draw a picture of the ladder leaning against the wall of the building and just touching the top of the fence.

If the ladder makes an angle 1.24 radians with the ground, touches the top of the fence and just reaches the wall, calculate the distance along the ladder from the ground to the top of the fence.

The distance along the ladder from the top of the fence to the wall is \_\_\_\_\_

Using these hints write a function  $L(x)$  which gives the total length of a ladder which touches the ground at an angle  $x$ , touches the top of the fence and just reaches the wall.

$L(x) =$  \_\_\_\_\_ .  
 Use this function to find the length of the shortest ladder which will clear the fence.

The length of the shortest ladder is \_\_\_\_\_ feet.

7.(1 pt) A rancher wants to fence in an area of 1500000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use? \_\_\_\_\_

8.(1 pt) For the given cost function  $C(x) = 128\sqrt{x} + \frac{x^2}{343000}$  find

- a) The cost at the production level 1900 \_\_\_\_\_
- b) The average cost at the production level 1900 \_\_\_\_\_
- c) The marginal cost at the production level 1900 \_\_\_\_\_
- d) The production level that will minimize the average cost. \_\_\_\_\_
- e) The minimal average cost. \_\_\_\_\_

9.(1 pt) For the given cost function  $C(x) = 14400 + 600x + x^2$  find:  
 a) The cost at the production level 1400 \_\_\_\_\_  
 b) The average cost at the production level 1400 \_\_\_\_\_  
 c) The marginal cost at the production level 1400 \_\_\_\_\_  
 d) The production level that will minimize the average cost \_\_\_\_\_  
 e) The minimal average cost \_\_\_\_\_

10.(1 pt) For the given cost function  $C(x) = 4150 + 300x + 0.2x^2$  and the demand function  $p(x) = 900$ . Find the production level that will maximize profit.

11.(1 pt) A manufacture has been selling 1550 television sets a week at 360 each. A market survey indicates that for each 10 rebate offered to a buyer, the number of sets sold will increase by 100 per week.  
 a) Find the demand function  $p(x)$ , where  $x$  is the number of the television sets sold per week.

- $p(x) =$  \_\_\_\_\_
- b) How large rebate should the company offer to a buyer, in order to maximize its revenue? \_\_\_\_\_
- c) If the weekly cost function is  $93000 + 120x$ , how should it set the size of the rebate to maximize its profit? \_\_\_\_\_

12.(1 pt) A baseball team plays in he stadium that holds 66000 spectators. With the ticket price at 9 the average attendance has been 27000. When the price dropped to 8, the average attendance rose to 33000.  
 a) Find the demand function  $p(x)$ , where  $x$  is the number of the spectators. (assume  $p(x)$  is linear)  $p(x) =$  \_\_\_\_\_  
 b) How should be set a ticket price to maximize revenue? \_\_\_\_\_

13.(1 pt) The manager of a large apartment complex knows from experience that 120 units will be occupied if the rent is 320 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 5 dollar increase in rent. Similarly, one additional unit will be occupied for each 5 dollar decrease in rent. What rent should the manager charge to maximize revenue? \_\_\_\_\_

14.(1 pt) A Norman window has the shape of a semicircle atop a rectangle so that the diameter of the semicircle is equal to the width of the rectangle. What is the area of the largest possible Norman window with a perimeter of 50 feet? \_\_\_\_\_

15.(1 pt) Let  $Q = (0, 3)$  and  $R = (12, 7)$  be given points in the plane. We want to find the point  $P = (x, 0)$  on the  $x$ -axis such that the sum of distances  $PQ + PR$  is as small as possible. (Before proceeding with this problem, draw a picture!)

To solve this problem, we need to minimize the following function of  $x$ :

$f(x) =$  \_\_\_\_\_

over the closed interval  $[a, b]$  where  $a =$  \_\_\_\_\_ and  $b =$  \_\_\_\_\_ .

We find that  $f(x)$  has only one critical number in the interval at  $x =$  \_\_\_\_\_

where  $f(x)$  has value \_\_\_\_\_

Since this is smaller than the values of  $f(x)$  at the two endpoints, we conclude that this is the minimal sum of distances.

**16.**(1 pt) Centerville is the headquarters of Greedy Cablevision Inc. The cable company is about to expand service to two nearby towns, Springfield and Shelbyville. There needs to be cable connecting Centerville to both towns. The idea is to save on the cost of cable by arranging the cable in a Y-shaped configuration. Centerville is located at  $(12, 0)$  in the  $xy$ -plane,

Springfield is at  $(0, 11)$ , and Shelbyville is at  $(0, -11)$ . The cable runs from Centerville to some point  $(x, 0)$  on the  $x$ -axis where it splits into two branches going to Springfield and Shelbyville. Find the location  $(x, 0)$  that will minimize the amount of cable between the 3 towns and compute the amount of cable needed. Justify your answer.

To solve this problem we need to minimize the following function of  $x$ :

$f(x) =$  \_\_\_\_\_

We find that  $f(x)$  has a critical number at  $x =$  \_\_\_\_\_

To verify that  $f(x)$  has a minimum at this critical number we compute the second derivative  $f''(x)$  and find that its value at the critical number is \_\_\_\_\_ , a positive number.

Thus the minimum length of cable needed is \_\_\_\_\_