1.(1 pt) Match each of the following differential equations with a solution from the list below.

__ 1. $y'' + y = 0$
__ 2. $y'' - 9y' + 18y = 0$
__ 3. $y'' + 9y' + 18y = 0$
__ 4. $2x^2y'' + 3xy' = y$

A. $y = \frac{1}{x}$
B. $y = \cos(x)$
C. $y = e^{-3x}$
D. $y = e^{6x}$

2.(1 pt) Match each of the differential equation with its solution.

__ 1. $y'' + y = 0$
__ 2. $y'' + 11y' + 30y = 0$
__ 3. $xy' - y = x^2$
__ 4. $2x^2y'' + 3xy' = y$

A. $y = \sin(x)$
B. $y = e^{-6x}$
C. $y = x^{\frac{1}{2}}$
D. $y = 3x + x^2$

3.(1 pt) Match each differential equation to a function which is a solution.

FUNCTIONS
A. $y = 3x + x^2$,
B. $y = e^{-5x}$,
C. $y = \sin(x)$,
D. $y = x^{\frac{1}{2}}$,
E. $y = 2\exp(7x)$,

DIFFERENTIAL EQUATIONS
__ 1. $2x^2y'' + 3xy' = y$
__ 2. $xy' - y = x^2$
__ 3. $y'' + 8y' + 15y = 0$
__ 4. $y'' + y = 0$

4.(1 pt) Match the following differential equations with their solutions.
The symbols A, B, C in the solutions stand for arbitrary constants.
You must get all of the answers correct to receive credit.

__ 1. $\frac{d^2y}{dx^2} + 4y = 0$
__ 2. $\frac{dy}{dx} = \frac{-2xy}{x^2 - 2y^2}$
__ 3. $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 0$
__ 4. $\frac{dy}{dx} = 4xy$
__ 5. $\frac{dy}{dx} + 21x^2y = 21x^2$

A. $y = A\cos(2x) + B\sin(2x)$
B. $3yx^2 - 2y^3 = C$
C. $y = Ce^{-7x^3} + 1$
D. $y = Ae^{2x^2}$
E. $y = Ae^{-7x} + Bxe^{-7x}$

5.(1 pt)
Just as there are simultaneous algebraic equations (where a pair of numbers have to satisfy a pair of equations) there are systems of differential equations, (where a pair of functions have to satisfy a pair of differential equations).

Indicate which pairs of functions satisfy this system. It will take some time to make all of the calculations.

$$y_1' = y_2 \qquad y_2' = -y_1$$

- A. $y_1 = e^{4x}$ $\quad y_2 = e^{4x}$
- B. $y_1 = e^{-x}$ $\quad y_2 = e^{-x}$
- C. $y_1 = \sin(x) + \cos(x)$ $\quad y_2 = \cos(x) - \sin(x)$
- D. $y_1 = e^{x}$ $\quad y_2 = e^{x}$
- E. $y_1 = 2e^{-2x}$ $\quad y_2 = 3e^{-2x}$
- F. $y_1 = \sin(x)$ $\quad y_2 = \cos(x)$
- G. $y_1 = \cos(x)$ $\quad y_2 = -\sin(x)$

As you can see, finding all of the solutions, particularly of a system of equations, can be complicated and time consuming. It helps greatly if we study the structure of the family of solutions to the equations. Then if we find a few solutions we will be able to predict the rest of the solutions using the structure of the family of solutions.

6.(1 pt) It can be helpful to classify a differential equation, so that we can predict the techniques that might help us to find a function which solves the equation. Two classifications are the **order of the equation** – (what is the highest number of derivatives involved) and whether or not the equation is **linear** .

Linearity is important because the structure of the the family of solutions to a linear equation is fairly simple. Linear equations can usually be solved completely and explicitly.

Determine whether or not each equation is linear:

[?] 1. $y'' - y + t^2 = 0$
[?] 2. $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} = 1$
[?] 3. $y'' - y + y^2 = 0$
[?] 4. $\frac{d^2y}{dt^2} + \sin(t + y) = \sin t$

7.(1 pt) It is easy to check that for any value of c, the function $y = x^2 + \frac{c}{x^2}$ is solution of equation $xy' + 2y = 4x^2$, $(x > 0)$. Find the value of c for which the solution satisfies the initial condition $y(9) = 8$.

$c =$ ______

8.(1 pt) The functions

$$y = x^2 + \frac{c}{x^2}$$

are all solutions of equation:

$$xy' + 2y = 4x^2, \ (x > 0)$$

Find the constant c which produces a solution which also satisfies the initial condition $y(1) = 8$.

$c =$ ______

9.(1 pt) It is easy to check that for any value of c, the function $y = ce^{-2x} + e^{-x}$ is solution of equation $y' + 2y = e^{-x}$. Find the value of c for which the solution satisfies the initial condition $y(-1) = 3$.

$c =$ ______

10.(1 pt) The family of functions $y = ce^{-2x} + e^{-x}$ is solution of the equation

$$y' + 2y = e^{-x}$$

Find the constant c which defines the solution which also satisfies the initial condition $y(-1) = 6$.

$c =$ ______

11.(1 pt) Find the two values of k for which $y(x) = e^{kx}$ is a solution of the differential equation
$y'' - 15y' + 54y = 0$.

smaller value = ______
larger value = ______

12.(1 pt) Some curves in the first quadrant have equations
$y = A\exp(7x)$, where A is a positive constant.
Different values of A give different curves. The curves form a family, F.
Let $P = (1, 6)$. Let C be the member of the family F that goes through P.
A. Let $y = f(x)$ be the equation of C. Find $f(x)$.
$f(x) =$ ______

B. Find the slope at P of the tangent to C.
slope: ______
C. A curve D is perpendicular to C at P . What is the slope of the tangent to D at the point P ? slope: ______
D. Give a formula $g(y)$ for the slope at (x, y) of the member of F that goes through (x, y). The formula should not involve A or x.
$g(y) =$ ______
E. A curve which at each of its points is perpendicular to the member of the family F that goes through the point is called an orthogonal trajectory to F. Each orthogonal trajectory to F satisfies the differential equation
$\frac{dy}{dx} = -\frac{1}{g(y)}$, where $g(y)$ is the answer to part D.
Find a function $h(y)$ such that $x = h(y)$ is the equation of the orthogonal trajectory to F that passes through the point P.
$h(y) =$ ______

13.(1 pt) The solution of a certain differential equation is of the form
$y(t) = a\exp(2t) + b\exp(6t)$, where a and b are constants.
The solution has initial conditions $y(0) = 2$ and $y'(0) = 2$.
Find the solution by using the initial conditions to get linear equations for a and b.

$y(t) =$ ______