1.(1 pt) Another "realistic" problem:

The following problem is similar to the problem in an earlier assignment about the bank account growing with periodic deposits. The basic procedure for this problem is not too hard, but getting details of the calculation correct is NOT easy, and may take some time.

A ping-pong ball is caught in a vertical plexiglass column in which the air flow alternates sinusoidally with a period of 60 seconds. The air flow starts with a maximum upward flow at the rate of $3.1m/s$ and at $t = 30$ seconds the flow has a minimum (upward) flow of rate of $-1.5m/s$. (To make this clear: a flow of $-5m/s$ upward is the same as a flow downward of $5m/s$.

The ping-pong ball is subjected to the forces of gravity $(-mg)$ where $g = 9.8m/s^2$ and forces due to air resistance which are equal to k times the apparent velocity of the ball through the air.

What is the average velocity of the air flow? You can average the velocity over one period or over a very long time – the answer should come out about the same – right?

____________ . (Include units).

Write a formula for the velocity of the air flow as a function of time.

$A(t) =$____________

Write the differential equation satisfied by the velocity of the ping-pong ball (relative to the fixed frame of the plexiglass tube.) The formulas should not have units entered, but use units to trouble shoot your answers. Your answer can include the parameters m - the mass of the ball and k the coefficient of air resistance, as well as time t and the velocity of the ball v. (Use just v, not v(t) the latter confuses the computer.)

$v'(t) =$

____

Use the method of undetermined coefficients to find one periodic solution to this equation:

$v(t) =$

____

Find the amplitude and phase shift of this solution. You do not need to enter units.

$v(t) =$ ______ cos(______ $*t-$ ______)

Find the general solution, by adding on a solution to the homogeneous equation. Notice that all of these solutions tend towards the periodically oscillating solution. This is a generalization of the notion of stability that we found in autonomous differential equations.

Calculate the specific solution that has initial conditions $t = 0$ and $w(0) = 2.7$.

$w(t) =$

____

Think about what effect increasing the mass has on the amplitude, on the phase shift? Does this correspond with your expectations?

2.(1 pt) A steel ball weighing 128 pounds is suspended from a spring. This stretches the spring $\frac{128}{401}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 8 feet per second.
The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second) .

Suppose that after t seconds the ball is y feet below its rest position. Find y in terms of t. (Note that this means that the postiive direction for y is down.)

$y =$

____

Take as the gravitational acceleration 32 feet per second per second.

3.(1 pt) A hollow steel ball weighing 4 pounds is suspended from a spring. This stretches the spring $\frac{1}{5}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 2 feet per second. The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second) .

Suppose that after t seconds the ball is y feet below its rest position. Find y in terms of t. (Note that the positive direction is down.)

Take as the gravitational acceleration 32 feet per second per second.

$y =$

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4.(1 pt) This problem is an example of critically damped harmonic motion.

A hollow steel ball weighing 4 pounds is suspended from a spring. This stretches the spring $\frac{1}{8}$ feet.

The ball is started in motion from the equilibrium position with a downward velocity of 2 feet per second. The air resistance (in pounds) of the moving ball numerically equals 4 times its velocity (in feet per second) . Suppose that after t seconds the ball is y feet below its rest position. Find y in terms of t.

Take as the gravitational acceleration 32 feet per second per second. (Note that the positive y direction is down in this problem.)

$y =$

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