

1.(1 pt) Write the given second order equation as its equivalent system of first order equations.

$$u'' + 2u' + 8u = 0$$

Use v to represent the "velocity function", i.e. $v = u'(t)$. Use v and u for the two functions, rather than $u(t)$ and $v(t)$. (The latter confuses webwork. Functions like $\sin(t)$ are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

Now write the system using matrices:

$$\frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}$$

2.(1 pt) Write the given second order equation as its equivalent system of first order equations.

$$u'' + 6u' + 3u = 6\sin(3t) \quad u(1) = 0, u'(1) = 1.5$$

Use v to represent the "velocity function", i.e. $v = u'(t)$. Use v and u for the two functions, rather than $u(t)$ and $v(t)$. (The latter confuses webwork. Functions like $\sin(t)$ are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

Now write the system using matrices:

$$\frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} - \\ - \end{pmatrix}$$

and the initial value for the vector valued function is:

$$\begin{pmatrix} u(1) \\ v(1) \end{pmatrix} = \begin{pmatrix} - \\ - \end{pmatrix}$$

3.(1 pt) Write the given second order equation as its equivalent system of first order equations.

$$t^2 u'' - 3.5tu' + (t^2 + 7.5)u = -8\sin(3t)$$

Use v to represent the "velocity function", i.e. $v = u'(t)$. Use v and u for the two functions, rather than $u(t)$ and $v(t)$. (The latter confuses webwork. Functions like $\sin(t)$ are ok.)

$$u' = \underline{\hspace{2cm}}$$

$$v' = \underline{\hspace{2cm}}$$

Now write the system using matrices:

$$\frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} - \\ - \end{pmatrix}$$

Match the differential equations and their matrix function solutions:

It's good practice to multiply at least one matrix solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row or one column.

4.(1 pt) This problem is similar to problem 21 on page 346. Consult that page for the diagram. You will probably want to write the solution out first, before trying to enter the answers into the computer.

Consider two interconnected tanks as shown in Fig 7.1.6 on page 347. Tank 1 initially contains 20 L (liters) of water and 375 g of salt, while tank 2 initially contains 80 L of water and 195 g of salt. Water containing 50 g/L of salt is poured into tank 1 at a rate of 4 L/min while the mixture flowing into tank 2 contains a salt concentration of 15 g/L of salt and is flowing at the rate of 1 L/min. The two connecting tubes have a flow rate of 6 L/min from tank 1 to tank 2; and of 2 L/min from tank 2 back to tank 1. Tank 2 is drained at the rate of 5 L/min.

You may assume that the solutions in each tank are thoroughly mixed so that the concentration of the mixture leaving any tank along any of the tubes has the same concentration of salt as the tank as a whole. (This is not completely realistic, but as in real physics, we are going to work with the approximate, rather than exact description. The 'real' equations of physics are often too complicated to even write down precisely, much less solve.)

How does the water in each tank change over time?

Let $p(t)$ and $q(t)$ be the amount of salt in g at time t in tanks 1 and 2 respectively. Write differential equations for p and q . (As usual, use the symbols p and q rather than $p(t)$ and $q(t)$.)

$$p' = \underline{\hspace{2cm}}$$

$$q' = \underline{\hspace{2cm}}$$

Give the initial values:

$$\begin{pmatrix} p(0) \\ q(0) \end{pmatrix} = \begin{pmatrix} - \\ - \end{pmatrix}$$

Show the equation that needs to be solved to find a constant solution to the differential equation:

$$\begin{pmatrix} - \\ - \end{pmatrix} = \begin{pmatrix} - & - \\ - & - \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

A constant solution is obtained if $p(t) = \underline{\hspace{2cm}}$ for all time t and $q(t) = \underline{\hspace{2cm}}$ for all time t .

5.(1 pt)

— 1.

$$y'(t) = \begin{pmatrix} -32 & 49 & -23 \\ -64 & 78 & -34 \\ 64 & -78 & 34 \end{pmatrix} y(t)$$

— 2.

$$y'(t) = \begin{pmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{pmatrix} y(t)$$

— 3.

$$y'(t) = \begin{pmatrix} -13 & -2 & 3 \\ -15 & -18 & 5 \\ -33 & -18 & 7 \end{pmatrix} y(t)$$

A.

$$y(t) = \begin{pmatrix} 0 & -1e^{26t} & 4e^{13t} \\ -4e^{13t} & -2e^{26t} & 5e^{13t} \\ 0 & 3e^{26t} & 1e^{13t} \end{pmatrix}$$

B.

$$y(t) = \begin{pmatrix} -1e^{0t} & 3e^{64t} & -3e^{16t} \\ -3e^{0t} & 4e^{64t} & -2e^{16t} \\ -5e^{0t} & -4e^{64t} & 2e^{16t} \end{pmatrix}$$

C.

$$y(t) = \begin{pmatrix} -1e^{-4t} & 2e^{-8t} & -1e^{-12t} \\ 0 & -2e^{-8t} & 5e^{-12t} \\ -3e^{-4t} & 2e^{-8t} & 3e^{-12t} \end{pmatrix}$$

6.(1 pt)

Match the differential equations and their vector valued function solutions:
 It will be good practice to multiply at least one solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row element.

— 1.

$$y'(t) = \begin{pmatrix} -97 & 33 & -5 \\ -140 & 84 & 35 \\ -4 & 15 & -8 \end{pmatrix} y(t)$$

— 2.

$$y'(t) = \begin{pmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{pmatrix} y(t)$$

— 3.

$$y'(t) = \begin{pmatrix} -86 & 218 & -160 \\ 73 & -49 & 80 \\ 111 & -138 & 165 \end{pmatrix} y(t)$$

A.

$$y(t) = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} e^{45t}$$

B.

$$y(t) = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix} e^{-21t}$$

C.

$$y(t) = \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} e^{13t}$$

7.(1 pt) Calculate the eigenvalues of this matrix:

[Note– you'll probably want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues. You can use the web version at [xFunctions](#)

If you select the "integral curves utility" from the main menu, will also be able to plot the integral curves of the associated differential equations.]

$$A = \begin{pmatrix} 8 & 8 \\ -24 & 40 \end{pmatrix}$$

smaller eigenvalue= _____
 associated eigenvector = (_____ , _____)
 larger eigenvalue = _____
 associated, eigenvector = (_____ , _____)

If $y' = Ay$ is a differential equation, how would the solution curves behave?

- A. The solution curves would race towards zero and then veer away towards infinity. (Saddle)

- B. All of the solution curves would run away from 0. (Unstable node)
- C. The solution curves converge to different points
- D. All of the solutions curves would converge towards 0. (Stable node)

8.(1 pt) Calculate the eigenvalues of this matrix:

[Note– you'll probably want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues. You can use the web version at [xFunctions](#)

If you select the "integral curves utility" from the main menu, will also be able to plot the integral curves of the associated differential equations.]

$$A = \begin{pmatrix} 26 & -20 \\ 10 & -19 \end{pmatrix}$$

smaller eigenvalue= _____
 associated eigenvector = (_____ , _____)
 larger eigenvalue = _____
 associated, eigenvector = (_____ , _____)

If $y' = Ay$ is a differential equation, how would the solution curves behave?

- A. All of the solutions curves would converge towards 0. (Stable node)
- B. The solution curves converge to different points

- C. All of the solution curves would run away from 0. (Unstable node)
 - D. The solution curves would race towards zero and then veer away towards infinity. (Saddle)
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