
1.(1 pt) This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can't be graded by WeBWorK, but is to be handed in at the first class after the due date.

A. State the uniqueness property of the fundamental theorem.

B. Show directly using the differential equation, that if $y_1(t)$ is a solution to the differential equation $y'(t) = y(t)$, then $y_2(t) = y_1(t+a)$ is also a solution to the differential equation. (You will need to use the known facts about y_1 to calculate that $y_2'(t) = y_2(t)$). (We know that the solution is the exponential function, but you will not need to use this fact.)

C. Describe the relationship between the graphs of y_1 and y_2 and using a sketch of the direction field explain why it is obvious that if y_1 is a solution then y_2 has to be a solution also.

D. Describe in words why if $y_1(t)$ is any solution to the differential equation $y' = f(y)$ then $y_2(t) = y_1(t+a)$ is also a solution.

E. Show that if $y_1(t)$ solves $y'(t) = y(t)$, then $y_2(t) = Ay_1(t)$ also solves the same equation.

F. Suppose that $y_1(t)$ solves $y'(t) = y(t)$ and $y(0) = 1$. (Such a solution is guaranteed by the fundamental theorem.). Let $y_2(t) = y_1(t+a)$ and let $y_3(t) = y_1(a)y_1(t)$. Calculate the values $y_2(0)$ and $y_3(0)$. Use the uniqueness property to show that $y_2(t) = y_3(t)$ for all t.

G. Explain how this proves that any solution to $y' = y$ must be a function which obeys the law of exponents.

H. Let $z = x+iy$. Define $\exp(z)$ (or e^z) using a Taylor series. Show that if $z = x+iy$ is a constant, then

$$\frac{d}{dt}\exp(tz) = z\exp(tz)$$

by differentiating the power series.

I. Use your earlier results to show that $\exp(z+w) = \exp(z)\exp(w)$. This method of checking the law of exponents is MUCH easier than expanding the power series.

You can find a direction field plotter **here** or at the
direction field plotter page . Choose ”integral curves utility” from the ”main screen” menu of xFunctions to get to the phaseplane plotter.

2.(1 pt) This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can't be graded by WeBWorK, but is to be handed in at the first class after the due date.

A. Using the same technique as in the previous problem show that if a function $y_1(t)$ satisfies: (1) $y_1(0)$ = 1 and (2) $y'(t) = y(t)$ then

$$(y_1(t))^r = y_1(rt)$$

B. Explain in words how this relates to another law of exponents.

You can find a direction field plotter htmlLink(”http://webwork.math.rochester.edu/mth163/phaseplaneplotters/launchXfunctions.html”, ”here”) or at the
htmlLink(”http://webwork.math.rochester.edu/mth163/phaseplaneplotters/”, ”direction field plotter page”). Choose ”integral curves utility” from the ”main screen” menu of xFunctions to get to the phaseplane plotter.

3.(1 pt) This problem involves using the uniqueness property (from the Fundamental Theorem of ordinary differential equations.) It can't be graded by WeBWorK, but is to be handed in at the first class after the due date.

Use the same ideas as in the previous problems.

A. Suppose that $y_1(t)$ satisfies the equation $y'' + y = 0$ and $y_1(0) = 0$and $y_1'(0) = 1$. Such a function exists because of the fundamental theorem. (We all know that it is $sin(t)$, but you should not use that fact in answering the questions below.)

Show that $y_2(t) = y_1'(t)$ also satisfies the equation $y'' + y = 0$ and that $y_2(0) = 1$ and $y_2'(0) = 0$.

B. If $y_3(t) = y_2'(t)$ show, using the uniqueness property, that $y_3(t) = -y_1(t)$

C. State the uniqueness property for solutions to second order differential equations (or equivalently to a system of two first order differential equations).

D. Use the uniqueness property to show that $y_1(t+a) = y_1'(a)y_1(t) + y_1(a)y_2(t) = y_2(a)y_1(t) + y_1(a)y_2(t)$

The formulas for the sin of sums of angles can be calculated completely from the one fact that it satisfies a differential equation. This is a general fact. Any solution of a differential equation has the potential for obeying certain "laws" which are dictated by the differential equation.