
1.(1 pt)

Let $P(x)$ be the statement “ the word x contains the letter ‘a’ ”. What are the truth values of the following?

- 1. $P(\text{lemon})$
 - 2. $P(\text{orange})$
 - 3. $P(\text{false})$
 - 4. $P(\text{true})$
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2.(1 pt)

Determine the truth value of the following statements if the universe of discourse of each variable is the set of real numbers.

- 1. $\exists x \exists y (x + y \neq y + x)$
- 2. $\forall x \exists y ((x + y = 2) \wedge (2x - y = 1))$
- 3. $\forall x \exists y (x + y = 1)$
- 4. $\exists x \forall y (xy = 0)$
- 5. $\forall x \exists y (x = y^2)$
- 6. $\forall x \neq 0 \exists y (xy = 1)$

- 7. $\exists x \exists y ((x + 2y = 2) \wedge (2x + 4y = 5))$
 - 8. $\exists x (x^2 = -1)$
 - 9. $\exists x (x^2 = 2)$
 - 10. $\forall x \exists y (x^2 = y)$
 - 11. $\forall x \forall y \exists z (z = \frac{x+y}{2})$
 - 12. $\exists x \forall y \neq 0 (xy = 1)$
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3.(1 pt)

The notation

 $\exists! x P(x)$

denotes the proposition

“ There exists a unique x such that $P(x)$ is true. ”

If the universe of discourse is the set of integers, what are the truth values of the following?

- 1. $\exists! x (x > 1)$
- 2. $\exists! x (x^2 = 1)$
- 3. $\exists! x (x + 3 = 2x)$
- 4. $\exists! x (x = x + 1)$