

1.(1 pt)

Determine whether f is a function from \mathbb{Z} to \mathbb{R} .

Enter "Y" for yes and "N" for no.

- 1. $f(n) = \frac{1}{n^2-16}$
- 2. $f(n) = \pm n$
- 3. $f(n) = \sqrt{n^2+4}$
- 4. $f(n) = 1/(n^2+6)$

2.(1 pt) Find the following values.

(a) $\lfloor 1.1 \rfloor$

(b) $\lceil 1.1 \rceil$

(c) $\lfloor -0.1 \rfloor$

(d) $\lceil -0.1 \rceil$

(e) $\lceil 2.99 \rceil$

(f) $\lceil -2.99 \rceil$

(g) $\lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil$

(h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$

3.(1 pt)

Determine if each of the following functions from $\{a, b, c, d\}$ to itself is one-to-one and/or onto.

Check ALL correct answers.

(a) $f(a) = b, f(b) = a, f(c) = b, f(d) = c$

- A. one-to-one.
- B. onto.
- C. neither one-to-one nor onto.

$f(a) = d, f(b) = a, f(c) = c, f(d) = b$

- A. neither one-to-one nor onto.
- B. onto.
- C. one-to-one.

$f(a) = c, f(b) = d, f(c) = a$

- A. one-to-one.
- B. neither one-to-one nor onto.
- C. onto.

4.(1 pt) Given that $f(x) = 7x^2 + 1$ and $g(x) = 6x + 7$ are functions from \mathbb{R} to \mathbb{R} , find

(a) $f \circ g$.

(b) $g \circ f$.

5.(1 pt) Find the number of bytes required to encode n bits of data where n equals:

- (a) 4. _____
- (b) 10. _____
- (c) 500. _____
- (d) 3000. _____

6.(1 pt)

Determine if each of the following functions is $O(x^2)$.

Answer Y for yes and N for no.

- 1. $f(x) = x \log(x)$
- 2. $f(x) = 2^x$
- 3. $f(x) = \log(2^x)$
- 4. $f(x) = \lfloor x \rfloor \cdot \lceil x \rceil$
- 5. $f(x) = x^2 + 1000$
- 6. $f(x) = \frac{x^4}{2}$
- 7. $f(x) = 17x + 11$

7.(1 pt) Find the least integer n such that $f(x)$ is $O(x^n)$ for each of the following functions:

(a) $f(x) = 2x^2 + x^8 \log(x)$ _____

(b) $f(x) = 3x^9 + (\log x)^4$ _____

(c) $f(x) = \frac{x^4 + x^2 + 1}{x^4 + 1}$ _____

(d) $f(x) = \frac{x^3 + 5 \log(x)}{x^4 + 1}$ _____

8.(1 pt)

Check the answer that best describes the relationship between $f(x)$ and x .

(For example if $f(x)$ is $\Theta(x)$ check that as your answer and not $O(x)$ or $\Omega(x)$ even though these are true also in this case.)

(a) $f(x) = 10$ is

- A. $O(x)$
- B. $\Theta(x)$
- C. $\Omega(x)$

(b) $f(x) = 3x + 7$ is

- A. $O(x)$
- B. $\Omega(x)$
- C. $\Theta(x)$

(c) $f(x) = x^2 + x + 1$ is

- A. $\Theta(x)$
- B. $O(x)$
- C. $\Omega(x)$

(d) $f(x) = 5 \log(x)$ is

- A. $\Theta(x)$
- B. $\Omega(x)$
- C. $O(x)$

(e) $f(x) = \lfloor x \rfloor$ is

- A. $\Omega(x)$
- B. $O(x)$
- C. $\Theta(x)$

9.(1 pt) In this problem it will be useful to recall the following properties of logarithms: $\log(xy) = \log(x) + \log(y)$ and $\log(x^a) = a \log(x)$.

Find the least integer k such that $f(n)$ is $O(n^k)$ for each of the following functions:

- (a) $f(n) = n \log(4^n)$ _____
(b) $f(n) = 1^8 + 2^8 + \dots + n^8$ _____
(c) $f(n) = \log(n!)$ _____
(d) $f(n) = \frac{\log(n^n)}{n^2+1}$ _____