

1.(1 pt)

Are the following integers primes? Enter "1" for a prime and "0" otherwise.

- \_\_\_ 1. 153
- \_\_\_ 2. 52
- \_\_\_ 3. 187
- \_\_\_ 4. 146
- \_\_\_ 5. 81
- \_\_\_ 6. 82

2.(1 pt) Find the prime factorization of the following numbers: (write  $p^0$  if a prime does not appear in the given number.)

$$150 = 2^a 3^b 5^c 7^d 11^e 13^f 17^g 19^h$$

where

$$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}} \quad d = \underline{\hspace{1cm}} \quad e = \underline{\hspace{1cm}} \quad f = \underline{\hspace{1cm}} \quad g = \underline{\hspace{1cm}} \\ h = \underline{\hspace{1cm}}$$

$$140 = 2^a 3^b 5^c 7^d 11^e 13^f 17^g 19^h$$

where

$$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}} \quad d = \underline{\hspace{1cm}} \quad e = \underline{\hspace{1cm}} \quad f = \underline{\hspace{1cm}} \quad g = \underline{\hspace{1cm}} \\ h = \underline{\hspace{1cm}}$$

$$2992 = 2^a 3^b 5^c 7^d 11^e 13^f 17^g 19^h$$

where

$$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}} \quad d = \underline{\hspace{1cm}} \quad e = \underline{\hspace{1cm}} \quad f = \underline{\hspace{1cm}} \quad g = \underline{\hspace{1cm}} \\ h = \underline{\hspace{1cm}}$$

$$454597 = 2^a 3^b 5^c 7^d 11^e 13^f 17^g 19^h$$

where

$$a = \underline{\hspace{1cm}} \quad b = \underline{\hspace{1cm}} \quad c = \underline{\hspace{1cm}} \quad d = \underline{\hspace{1cm}} \quad e = \underline{\hspace{1cm}} \quad f = \underline{\hspace{1cm}} \quad g = \underline{\hspace{1cm}} \\ h = \underline{\hspace{1cm}}$$

3.(1 pt) The value of the Euler  $\phi$  function ( $\phi$  is the Greek letter phi) at the positive integer  $n$  is defined to be the number of positive integers less than or equal to  $n$  that are relatively prime to  $n$ . For example for  $n=14$ , we have  $\{1, 3, 5, 9, 11, 13\}$  are the positive integers less than or equal to 14 which are relatively prime to 14. Thus  $\phi(14) = 6$ . Find:

- $\phi(5)$  \_\_\_\_\_
- $\phi(25)$  \_\_\_\_\_
- $\phi(15)$  \_\_\_\_\_
- $\phi(45)$  \_\_\_\_\_

4.(1 pt) What are the greatest common divisors of the following pairs of integers?

- (a)  $2^3 * 3^3 * 5^1$  and  $2^4 * 3^4 * 5^1$  \_\_\_\_\_
- (b)  $2^{11} * 3 * 5 * 7 * 11 * 13$  and  $13^{11} * 3^4 * 2^4 * 17$  \_\_\_\_\_
- (c)  $2^3 * 7$  and  $5^3 * 13$  \_\_\_\_\_

5.(1 pt) Find the memory locations which are assigned by the hashing function  $h(k) = k \bmod 101$  to the records of students with the following Social Security numbers:

(Note enter the canonical representative for the answer modulo 101, thus your answer should be an integer between 0 and 100 inclusive for each part.)

$$261573373 \quad \underline{\hspace{1cm}} \quad 711300711 \quad \underline{\hspace{1cm}} \quad 328848800 \quad \underline{\hspace{1cm}} \quad 657810156$$

6.(1 pt) Find the first few terms of the sequence of pseudorandom numbers generated using the linear congruential generator  $x_{n+1} = (6x_n + 7) \bmod 8$  with seed  $x_0 = 2$ ?

$$x_1 = \underline{\hspace{1cm}} \quad x_2 = \underline{\hspace{1cm}} \quad x_3 = \underline{\hspace{1cm}} \quad x_4 = \underline{\hspace{1cm}} \quad x_5 = \underline{\hspace{1cm}} \quad x_6 = \underline{\hspace{1cm}}$$

Now find the first few terms of the sequence of pseudorandom numbers generated using the linear congruential generator  $x_{n+1} = (8x_n + 3) \bmod 9$  with the seed  $x_0 = 1$ ?

$$x_1 = \underline{\hspace{1cm}} \quad x_2 = \underline{\hspace{1cm}} \quad x_3 = \underline{\hspace{1cm}} \quad x_4 = \underline{\hspace{1cm}} \quad x_5 = \underline{\hspace{1cm}} \quad x_6 = \underline{\hspace{1cm}}$$

7.(1 pt) Encrypt the message HALT by translating the letters into numbers

(via  $A = 0, B = 1, C = 2, D = 3, E = 4, F = 5, G = 6, H = 7, I = 8, J = 9, K = 10, L = 11, M = 12, N = 13, O = 14, P = 15, Q = 16, R = 17, S = 18, T = 19, U = 20, V = 21, W = 22, X = 23, Y = 24, Z = 25$ )

and then applying the encryption function given, and then translating the numbers back into letters.

- (a)  $f(p) = (p + 3) \bmod 26$  \_\_\_\_\_
- (b)  $f(p) = (p + 10) \bmod 26$  \_\_\_\_\_
- (c)  $f(p) = (4p + 4) \bmod 26$  \_\_\_\_\_

8.(1 pt) Decrypt the following messages encrypted using the Caesar cipher:

$f(p) = (p + 3) \bmod 26$   
 Alphabet: A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W,X,Y,Z

- (a) EOXH MHDQV \_\_\_\_\_
- (b) HDW GLP VXP \_\_\_\_\_
- (c) HFZBSODU \_\_\_\_\_

9.(1 pt) Books are identified by an International Standard Book Number (ISBN), a 10-digit code  $x_1x_2 \dots x_{10}$ , assigned by the publisher. These 10 digits consist of blocks identifying the language, the publisher, the number assigned to the book by its publishing company, and finally, a 1-digit check digit that is either a digit or the letter X (used to represent 10). This check digit is selected so that  $\sum_{i=1}^{10} ix_i \equiv 0 \pmod{11}$  and is used to detect errors in individual digits and transposition of digits.

- (a) The ISBN for a book is 2 – 45 – 207476 – Q where Q is the check digit. What is Q? \_\_\_\_\_
- (b) The ISBN of another book is 9 – 755 – 60M37 – 0. Find the digit M. \_\_\_\_\_

10.(1 pt) Covert the following integers from decimal notation to binary notation.

(Do not put extra zeros in front of your binary notation or it might confuse WebWorK. So write 101 versus 0101 etc.)

- (a) 500 \_\_\_\_\_  
(b) 1500 \_\_\_\_\_  
(c) 184601 \_\_\_\_\_

**11.**(1 pt) Convert the following integers from binary notation to decimal notation:

- (a) 100000010 \_\_\_\_\_  
(b) 100000010 \_\_\_\_\_

**12.**(1 pt) Convert each of the following integers from binary notation to octal and hexadecimal notation.

- (a) 1100111010  
octal: \_\_\_\_\_ hexadecimal: \_\_\_\_\_  
(b) 1110000000  
octal: \_\_\_\_\_ hexadecimal: \_\_\_\_\_  
(c) 10000110001  
octal: \_\_\_\_\_ hexadecimal: \_\_\_\_\_

**13.**(1 pt) One's complement representations of integers are used to simplify computer arithmetic. To represent positive and negative integers with absolute value less than  $2^{n-1}$ , a total of  $n$  bits is used. The leftmost bit is used to represent the sign: this bit is 0 for positive integers and 1 for negative integers.

For positive integers, the remaining bits are identical to the binary representation of the integer. Thus using 4 bits, the One's complement representation of 5 is 0101.

For negative integers, the remaining bits are found by first finding the binary representation of the absolute value of the integer and then taking the complement of each of these bits in the binary representation. (Thus 1's and 0's get switched)

Thus for example using 4 bits, the One's complement representation of -5 is 1010, where the initial 1 is to indicate the sign, and the subsequent 010 is the complement of 101 the binary representation of 5.

Find the One's Complement representations, using bit strings of length six, of the following integers. (Your answers must be bit strings of length 6):

- (a) 27 \_\_\_\_\_  
(b) 28 \_\_\_\_\_  
(c) -23 \_\_\_\_\_  
(d) -29 \_\_\_\_\_

**14.**(1 pt) A Cantor expansion is a sum of the form

$$a_n n! + a_{n-1} (n-1)! + \dots + a_2 2! + a_1 1!$$

where  $0 \leq a_i \leq i$  are integers for  $i = 1, 2, \dots, n$ .

For example, the Cantor expansion of 17 is  $17 = 2(3!) + 2(2!) + 1(1!)$ . Note that  $17 = 1(3!) + 5(2!) + 1(1!)$  is correct but is not a valid Cantor expansion of 17 since we insist the coefficient of  $n!$  is no more than  $n$ . Thus the coefficient of  $2!$  should be a 0, 1 or 2 for example.

Complete the following Cantor expansions of:

- (a)  $21 = \underline{\hspace{1cm}} (3!) + \underline{\hspace{1cm}} (2!) + \underline{\hspace{1cm}} (1!)$   
(b)  $414 = \underline{\hspace{1cm}} (5!) + \underline{\hspace{1cm}} (4!) + \underline{\hspace{1cm}} (3!) + \underline{\hspace{1cm}} (2!) + \underline{\hspace{1cm}} (1!)$   
(c)  $86 = \underline{\hspace{1cm}} (4!) + \underline{\hspace{1cm}} (3!) + \underline{\hspace{1cm}} (2!) + \underline{\hspace{1cm}} (1!)$