1.(1 pt)
Which rule of inference is used in each of the following arguments? Check the correct answers.

1. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

- A. Simplification.
- B. Conjuction.
- C. Modus ponens.
- D. Hypothetical syllogism.
- E. Addition.
- F. Modus tollens.
- G. Disjunctive syllogism.

2. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

- A. Simplification.
- B. Hypothetical syllogism.
- C. Modus tollens.
- D. Conjunction.
- E. Disjunctive syllogism.
- F. Modus ponens.
- G. Addition.

3. If it is rainy, then the pool will be closed. It is rainy. Therfore, the pool is closed.

- A. Modus ponens.
- B. Conjunction.
- C. Hypothetical syllogism.
- D. Disjunctive syllogism.
- E. Modus tollens.
- F. Simplification.
- G. Addition.

4. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

- A. Simplification.
- B. Addition.
- C. Conjunction.
- D. Modus tollens.
- E. Modus ponens.
- F. Disjunctive syllogism.
- G. Hypothetical syllogism.

2.(1 pt) On a 8×8 chessboard, the squares are colored alternately white and black. Thus there are ____ white squares and ___ black squares. Each row/column of the chessboard has ___ squares.
It is thus possible to tile this chessboard with dominoes (1 x 2 pieces) by laying say 4 dominoes per column. (tile means lay the dominoes, so that they cover the chessboard, no two dominoes overlapping.)

Now suppose we remove two squares from the chessboard, from DIAGONALLY opposite corners. Suppose one of the squares we remove is white. Now there are ____ white squares left and ______________________ black squares left.

Q: Is it possible to cover the modified chessboard (with the two diagonally opposite corners removed) with dominoes? Why?

- A. No. Since the total number of remaining squares on the chessboard is odd and every domino covers 2 squares and hence can only be used to tile a region with an even number of squares.
- B. No. Since every time we lay down a domino it covers one white square and one black square. Thus since the number of white squares is not equal to the number of black squares on the modified chessboard, it is impossible.
- C. Yes. It is possible to tile the modified chessboard by placing dominoes, alternating between horizontal and vertical placements in a suitable way.
- D. Yes. Since there are an equal number of white and black squares remaining on the modified chessboard, one can tile the modified chessboard with dominoes each covering one white and one black square.

3.(1 pt) For n a nonnegative integer, either $n \equiv 0$ mod 3 or $n \equiv 1$ mod 3 or $n \equiv 2$ mod 3. In each case, fill out the following table with the canonical representatives modulo 3 of the expressions given:

n mod 3	n^3 mod 3	$2n$ mod 3	n^3+2n mod 3
0	______	______	________
1	______	______	________
2	______	______	________

From this, we can conclude:

- A. Since $n^3+2n \not\equiv 0$ mod 3 for all n, we conclude that 3 does not necessarily divide n^3+2n for all nonnegative integers n.
- B. Since $n^3+2n \equiv 0$ mod 3 for all n, we conclude that 3 divides n^3+2n for any nonnegative integer n.

4.(1 pt) Find f(1), f(2), f(3) and f(4) if f(n) is defined recursively by $f(0)=3$ and for $n=0,1,2,\ldots$ by:
(a) $f(n+1)=-2f(n)$
$f(1)=$ ____ $f(2)=$ ____ $f(3)=$ ____ $f(4)=$ ____
(b) $f(n+1)=3f(n)+7$
$f(1)=$ ____ $f(2)=$ ____ $f(3)=$ ____ $f(4)=$ ____
(b) $f(n+1)=f(n)^2-3f(n)-4$
$f(1)=$ ____ $f(2)=$ ____ $f(3)=$ ____ $f(4)=$ ____

5.(1 pt) Consider the following inductive definition of a version of Ackermann's function:

$$A(m,n) = \begin{cases} 2n \text{ if } m = 0 \\ 0 \text{ if } m \geq 1 \text{ and } n = 0 \\ 2 \text{ if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m, n-1)) \text{ if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

Find the following values of the Ackermann's function:

$A(2,0) =$ ____ $A(2,3) =$ ____ $A(1,0) =$ ____ $A(1,3) =$ ____

$A(0,1) =$ ____ $A(3,3) =$ ____