

**1.**(1 pt) (a) A particular brand of shirt comes in 14 colors, has a male version and a female version, and comes in 2 sizes for each sex. How many different types of this shirt are made?

(b) How many bit strings of length 8 are there?

(c) How many bit strings of length 8 or less are there? (Count the empty string of length zero also.)

(d) How many strings of 3 lower case English letters are there that have the letter x in them somewhere? Here strings may use the same letter more than once. (Hint: It might be easier to first count the strings that don't have an x in them.)

**2.**(1 pt) Find how many positive integers with exactly four decimal digits, that is, positive integers between 1000 and 9999 inclusive, have the following properties:

(a) are divisible by 5 but not by 7.

(b) are divisible by 5.

(c) are not divisible by either 5 or 7.

(d) are divisible by 7.

**3.**(1 pt) How many strings of four decimal digits (Note there are 10 possible digits and a string can be of the form 0014 etc., i.e., can start with zeros.)

(a) begin and end with a 1?

(b) have exactly three digits which are 9s?

**4.**(1 pt) How many strings of five uppercase English letters are there

(a) that start and end with the letters BO (in that order), if letters can be repeated?

(b) that start or end with the letters BO (in the order), if letters can be repeated? (inclusive or)

(c) if no letter can be repeated?

(d) if letters can be repeated?

**5.**(1 pt) Solve the following two "union" type questions:

(a) How many bit strings of length 8 either begin with 1 0s or end with 2 1s? (inclusive or)

(b) Every student in a discrete math class is either a computer science or a mathematics major or is a joint major in these

two subjects. How many students are in the class if there are 36 computer science majors (including joint majors), 23 math majors (including joint majors) and 8 joint majors?

**6.**(1 pt) A bowl contains 8 red balls and 8 blue balls. A woman selects balls at random without looking at them.

(a) How many balls must she select (minimum) to be sure of having at least three blue balls? \_\_\_\_\_

(b) How many balls must she select (minimum) to be sure of having at least three balls of the same color? \_\_\_\_\_

**7.**(1 pt) This question concerns bit strings of length six. These bit strings can be divided up into four types depending on their initial and terminal bit. Thus the types are: 0XXXX0, 0XXXX1, 1XXXX0, 1XXXX1.

How many bit strings of length six must you select before you are sure to have at least 6 that are of the same type? (Assume that when you select bit strings you always select different ones from ones you have already selected.)

**8.**(1 pt) Find the value of each of the following quantities:

$C(12, 6) =$  \_\_\_\_\_

$C(6, 6) =$  \_\_\_\_\_

$C(6, 3) =$  \_\_\_\_\_

$C(12, 1) =$  \_\_\_\_\_

$C(6, 3) =$  \_\_\_\_\_

$C(8, 1) =$  \_\_\_\_\_

**9.**(1 pt) There are 7 different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

**10.**(1 pt) How many bit strings of length 8 have:

(a) Exactly three 0s? \_\_\_\_\_

(b) The same number of 0s as 1s? \_\_\_\_\_

(d) At least three 1s? \_\_\_\_\_

**11.**(1 pt) 13 players for a softball team show up for a game:

(a) How many ways are there to choose 10 players to take the field?

(b) How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

(c) Of the 13 people who show up, 3 are women. How many ways are there to choose 10 players to take the field if at least one of these players must be women?

**12.**(1 pt) Suppose that a department contains 8 men and 15 women. How many ways are there to form a committee with 6 members if it must have strictly more women than men?

**13.**(1 pt) How many ways are there to select 11 countries in the United Nations to serve on a council if 2 is selected from a

block of 54, 2 are selected from a block of 69 and 7 are selected from the remaining 66 countries?

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14.(1 pt) Find the coefficient of  $x^6$  in  $(1+x)^{10}$ .

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15.(1 pt) What is the coefficient of  $x^4y^{12}$  in the expansion of  $(1x-1y)^{16}$ ?

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16.(1 pt) Two six-sided dice are rolled (one red one and one green one). Some possibilities are (Red=1,Green=5) or (Red=2,Green=2) etc.  
(a) How many total possibilities are there?

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For the rest of the questions, we will assume that the dice are fair and that all of the possibilities in (a) are equally likely.

(b) What is the probability that the sum on the two dice comes out to be 9? (Remember the answer will be a ratio, the denominator of which will be your answer in (a).)

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(c) What is the probability that the sum on the two dice comes out to be 8?

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(d) What is the probability that the numbers on the two dice are equal?

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A card is selected at random from a standard 52-card deck.

(a) What is the probability that it is an ace? \_\_\_\_\_

(b) What is the probability that it is a heart? \_\_\_\_\_

(c) What is the probability that is an ace or a heart? \_\_\_\_\_

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18.(1 pt) A five-card poker hand is dealt at random from a standard 52-card deck.

Note the total number of possible hands is  $C(52,5)=2,598,960$ .

Find the probabilities of the following scenarios:

(a) What is the probability that the hand contains exactly one ace?

Answer= $\frac{\alpha}{C(52,5)}$ , where  $\alpha =$  \_\_\_\_\_

(b) What is the probability that the hand is a flush? (That is all the cards are of the same suit: hearts, clubs, spades or diamonds.)

Answer= $\frac{\beta}{C(52,5)}$ , where  $\beta =$  \_\_\_\_\_

(c) What is the probability that the hand is a straight flush?

Answer= $\frac{\gamma}{C(52,5)}$ , where  $\gamma =$  \_\_\_\_\_

What is the probability that a positive integer  $m$  in the range  $1 \leq m \leq 100$ , which is selected randomly, is divisible by 6?

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