1.(1 pt) You are given the four points in the plane $A = (1,-6)$, $B = (6,6)$, $C = (11,-8)$, and $D = (15,1)$. The graph of the function $f(x)$ consists of the three line segments AB, BC and CD. Find the integral $\int_1^{15} f(x)\,dx$ by interpreting the integral in terms of sums and/or differences of areas of elementary figures.
$\int_1^{15} f(x)\,dx =$ ______

2.(1 pt) Evaluate the integral below by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.

$$\int_{-4}^{4} \sqrt{16-x^2}dx$$

3.(1 pt) Use the Midpoint Rule to approximate

$$\int_{-1.5}^{4.5} x^3 dx$$

with $n = 6$.

4.(1 pt) Evaluate the integral by interpreting it in terms of areas. In other words, draw a picture of the region the integral represents, and find the area using high school geometry.

$$\int_0^3 |10x-10|dx$$

5.(1 pt) Use the Midpoint Rule to approximate the integral

$$\int_{-7}^{0} (-1x+8x^2)dx$$

with n=3.

6.(1 pt) Consider the integral

$$\int_0^3 (2x^2+2x+2)\,dx$$

(a) Find the Riemann sum for this integral using right endpoints and $n = 3$.
$R_3 =$ ______
(b) Find the Riemann sum for this same integral, using left endpoints and $n = 3$.
$L_3 =$ ______

7.(1 pt) Consider the integral

$$\int_2^6 \left(\frac{3}{x}+3\right)\,dx$$

(a) Find the Riemann sum for this integral using right endpoints and $n = 4$.

______

(b) Find the Riemann sum for this same integral, using left endpoints and $n = 4$

8.(1 pt) Let $\int_1^4 f(x)dx = 1$, $\int_1^2 f(x)dx = 7$, $\int_3^4 f(x)dx = 6$.
Find $\int_2^3 f(x)dx =$ ______
and $\int_3^2 (1f(x)-7)dx =$ ______

9.(1 pt)

$$\int_8^{18} f(x) - \int_8^{13} f(x) = \int_a^b f(x)$$

where $a =$ ______ and $b =$ ______

10.(1 pt) Consider the function $f(x) = -\frac{x^2}{3}+1$.

In this problem you will calculate $\int_0^3 (-\frac{x^2}{3}+1)\,dx$ by using the definition

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n} f(x_i)\Delta x\right]$$

The summation inside the brackets is R_n which is the Riemann sum where the sample points are chosen to be the right-hand endpoints of each sub-interval.

Calculate R_n for $f(x) = -\frac{x^2}{3}+1$ on the interval $[0,3]$ and write your answer as a function of n without any summation signs. You will need the summation formulas on page 381 of your textbook (page 364 in older texts).
$R_n =$ ______
$\lim_{n\to\infty} R_n =$ ______

11.(1 pt) The following sum

$$\frac{1}{1+\frac{7}{n}}\cdot\frac{7}{n}+\frac{1}{1+\frac{14}{n}}\cdot\frac{7}{n}+\frac{1}{1+\frac{21}{n}}\cdot\frac{7}{n}+\ldots+\frac{1}{1+\frac{7n}{n}}\cdot\frac{7}{n}$$

is a right Riemann sum for a certain definite integral

$$\int_1^b f(x)\,dx$$

using a partition of the interval $[1,b]$ into n subintervals of equal length.
Then the upper limit of integration must be: $b =$ ______
and the integrand must be the function $f(x) =$ ______

12.(1 pt) The following sum

$$\sqrt{9+\frac{4}{n}}\cdot\left(\frac{4}{n}\right)+\sqrt{9+\frac{8}{n}}\cdot\left(\frac{4}{n}\right)+\ldots+\sqrt{9+\frac{4n}{n}}\cdot\left(\frac{4}{n}\right)$$

is a right Riemann sum for the definite integral

$$\int_6^b f(x)\,dx$$

where $b =$ ______
and $f(x) =$ ______

It is also a Riemann sum for the definite integral

$$\int_9^c g(x)\,dx$$

where $c =$ ____________________

and $g(x) =$ ____________________

The limit of these Riemann sums as $n \to \infty$ is ____________

13.(1 pt) The following sum

$$\sqrt{16-\left(\tfrac{4}{n}\right)^2}\cdot\tfrac{4}{n}+\sqrt{16-\left(\tfrac{8}{n}\right)^2}\cdot\tfrac{4}{n}+\ldots+\sqrt{16-\left(\tfrac{4n}{n}\right)^2}\cdot\tfrac{4}{n}$$

is a right Riemann sum for the definite integral

$$\int_0^b f(x)\,dx$$

where $b =$ ____________________

and $f(x) =$ ____________________

The limit of these Riemann sums as $n \to \infty$ is ____________

14.(1 pt) Suppose $f(x)$ is continuous and decreasing on the closed interval $6 \le x \le 11$, that $f(6) = 7$, $f(11) = 4$ and that $\int_6^{11} f(x)\,dx = 20.875767$.

Then $\int_4^7 f^{-1}(x)\,dx =$ ____________________

15.(1 pt) Consider the function

$$f(x) = x^3 - 15x^2 + 96x + 10$$

By drawing a suitable picture, find a relation between the definite integrals $\int_1^2 f(x)\,dx$ and $\int_{92}^{150} f^{-1}(x)\,dx$. Use this relation to find the second of these two integrals

$\int_{92}^{150} f^{-1}(x)\,dx =$ ____________________

16.(1 pt) Estimate the area under the graph of $f(x) = x^2 + 2x$ from $x = 1$ to $x = 9$ using 4 approximating rectangles and left endpoints.

17.(1 pt) Evaluate the definite integral by interpreting it in terms of areas.

$$\int_3^9 (4x - 16)dx$$

18.(1 pt) Given that $1 \le f(x) \le 2$ for $-6 \le x \le 4$, use property 8 on page 387 to estimate the value of $\int_{-6}^4 f(x)dx$

____________ $\le \int_{-6}^4 f(x)dx \le$ ____________