

1.(1 pt) The joint probability density function of X and Y is given by

$$f(x,y) = c(y^2 - 196x^2)e^{-y}, \quad -\frac{y}{14} \leq x \leq \frac{y}{14}, \quad 0 < y < \infty$$

Find c and the expected value of X :

$c =$ _____
 $E(X) =$ _____

2.(1 pt) x and y are uniformly distributed over the interval $[0, 1]$. Find the probability that $|x - y|$, the distance between x and y , is less than 0.4.

3.(1 pt) A man and a woman agree to meet at a cafe about noon. If the man arrives at a time uniformly distributed between 11 : 35 and 12 : 15 and if the woman independently arrives at a time uniformly distributed between 11 : 50 and 12 : 50, what is the probability that the first to arrive waits no longer than 15 minutes?

4.(1 pt) Two points are selected randomly on a line of length 16 so as to be on opposite sides of the midpoint of the line. In other words, the two points X and Y are independent random variables such that X is uniformly distributed over $[0, 8]$ and Y is uniformly distributed over $(8, 16]$. Find the probability that the distance between the two points is greater than 2.

answer: _____

5.(1 pt) Let

$$f(x) = \begin{cases} cx^{10}y^2 & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the following:

(a) c such that $f(x,y)$ is a probability density function:

$c =$ _____

(b) Expected values of X and Y :

$E(X) =$ _____

$E(Y) =$ _____

(c) Are X and Y independent? (enter YES or NO) _____

6.(1 pt) Let A , B , and C be independent random variables, uniformly distributed over $[0, 6]$, $[0, 9]$, and $[0, 11]$ respectively. What is the probability that both roots of the equation $Ax^2 + Bx + C = 0$ are real? _____

7.(1 pt) Assume that the monthly worldwide average number of airplane crashes of commercial airlines is 2.2. What is the probability that there will be

(a) at most 2 such accidents in the next month? _____

(b) less than 4 such accidents in the next 2 months? _____

(c) exactly 6 such accidents in the next 4 months? _____

8.(1 pt) Andrew's bowling scores are approximately normally distributed with mean 100 and standard deviation 19,

while Leo's scores are normally distributed with mean 105 and standard deviation 24. If Andrew and Leo each bowl one game, then assuming that their scores are independent random variables, approximate the probability that the total of their scores is above 205.

9.(1 pt) The joint probability mass function of X and Y is given by

$$\begin{array}{lll} p(1,1) = 0.2 & p(1,2) = 0.05 & p(1,3) = 0.15 \\ p(2,1) = 0.1 & p(2,2) = 0.05 & p(2,3) = 0.05 \\ p(3,1) = 0.15 & p(3,2) = 0.15 & p(3,3) = 0.1 \end{array}$$

(a) Compute the conditional mass function of Y given $X = 1$:

$P(Y = 1|X = 1) =$ _____

$P(Y = 2|X = 1) =$ _____

$P(Y = 3|X = 1) =$ _____

(b) Are X and Y independent? (enter YES or NO) _____

(c) Compute the following probabilities:

$P(X + Y > 4) =$ _____

$P(XY = 2) =$ _____

$P(\frac{X}{Y} > 2) =$ _____

10.(1 pt) The joint probability mass function of X and Y is given by

$$\begin{array}{lll} p(1,1) = 0.15 & p(1,2) = 0.05 & p(1,3) = 0.15 \\ p(2,1) = 0.15 & p(2,2) = 0.05 & p(2,3) = 0.15 \\ p(3,1) = 0.15 & p(3,2) = 0.1 & p(3,3) = 0.05 \end{array}$$

Compute the following probabilities:

$P(X + Y > 3) =$ _____

$P(XY = 4) =$ _____

$P(\frac{X}{Y} > 1) =$ _____

11.(1 pt) The joint probability mass function of X and Y is given by

$$\begin{array}{lll} p(1,1) = 0.2 & p(1,2) = 0.05 & p(1,3) = 0.05 \\ p(2,1) = 0.1 & p(2,2) = 0.05 & p(2,3) = 0.1 \\ p(3,1) = 0.1 & p(3,2) = 0.05 & p(3,3) = 0.3 \end{array}$$

(a) Compute the conditional mass function of Y given $X = 1$:

$P(Y = 1|X = 1) =$ _____

$P(Y = 2|X = 1) =$ _____

$P(Y = 3|X = 1) =$ _____

(b) Are X and Y independent? (enter YES or NO) _____

12.(1 pt) Two points along a straight stick of length 42cm are randomly selected. The stick is then broken at those two points. Find the probability that all of the resulting pieces have length at least 3.5cm.

