

1.(1 pt) Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If either test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

- 1.  $\sum_{n=1}^{\infty} \frac{3n^6}{n^{10} + 4}$
- 2.  $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^6}$
- 3.  $\sum_{n=1}^{\infty} \frac{7n^{10} - n^7 + 3\sqrt{n}}{4n^{12} - n^6 + 6}$
- 4.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n}$
- 5.  $\sum_{n=1}^{\infty} \frac{3n^6}{n^7 + 4}$

2.(1 pt) Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If either test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

- 1.  $\sum_{n=1}^{\infty} \frac{7n^6}{n^7 + 5}$
- 2.  $\sum_{n=1}^{\infty} \frac{8n^{10} - n^7 + 7\sqrt{n}}{3n^{12} - n^6 + 6}$
- 3.  $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^6}$
- 4.  $\sum_{n=1}^{\infty} \frac{7n^6}{n^{10} + 5}$
- 5.  $\sum_{n=1}^{\infty} \frac{(\ln(n))^6}{n + 6}$

3.(1 pt) Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If at least one test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

- 1.  $\sum_{n=1}^{\infty} \frac{3n^4}{n^5 + 10}$

- 2.  $\sum_{n=1}^{\infty} \frac{3n^4}{n^6 + 10}$
- 3.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$
- 4.  $\sum_{n=1}^{\infty} \frac{4n^6 - n^5 + 3\sqrt{n}}{2n^8 - n^4 + 4}$
- 5.  $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^4}$

4.(1 pt) Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. If at least one test can be applied to the series, enter CONV if it converges or DIV if it diverges. If neither test can be applied to the series, enter NA. (Note: this means that even if you know a given series converges by some other test, but the comparison tests cannot be applied to it, then you must enter NA rather than CONV.)

- 1.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n}$
- 2.  $\sum_{n=1}^{\infty} \frac{(\ln(n))^6}{n + 7}$
- 3.  $\sum_{n=1}^{\infty} \frac{\cos(n)\sqrt{n}}{3n + 7}$
- 4.  $\sum_{n=1}^{\infty} \frac{3n^5}{n^8 + 7}$
- 5.  $\sum_{n=1}^{\infty} \frac{3n^5}{n^6 + 7}$

5.(1 pt) Each of the following statements is an attempt to show that a given series is convergent or not using the Comparison Test (NOT the Limit Comparison Test.) For each statement, enter C (for "correct") if the argument is valid, or enter I (for "incorrect") if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

- 1. For all  $n > 2$ ,  $\frac{1}{n^2-5} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{1}{n^2-5}$  converges.
- 2. For all  $n > 2$ ,  $\frac{n}{n^3-9} < \frac{2}{n^2}$ , and the series  $2\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{n^3-9}$  converges.
- 3. For all  $n > 1$ ,  $\frac{\ln(n)}{n^2} < \frac{1}{n^{1.5}}$ , and the series  $\sum \frac{1}{n^{1.5}}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.
- 4. For all  $n > 2$ ,  $\frac{\ln(n)}{n} > \frac{1}{n}$ , and the series  $\sum \frac{1}{n}$  diverges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n}$  diverges.
- 5. For all  $n > 1$ ,  $\frac{1}{n \ln(n)} < \frac{2}{n}$ , and the series  $2\sum \frac{1}{n}$  diverges, so by the Comparison Test, the series  $\sum \frac{1}{n \ln(n)}$  diverges.

- 6. For all  $n > 1$ ,  $\frac{n}{5-n^3} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{1}{5-n^3}$  converges.

**6.(1 pt)** Each of the following statements is an attempt to show that a given series is convergent or not using the Comparison Test (NOT the Limit Comparison Test.) For each statement, enter C (for "correct") if the argument is valid, or enter I (for "incorrect") if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

- 1. For all  $n > 2$ ,  $\frac{1}{n^2-1} < \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{1}{n^2-1}$  converges.
- 2. For all  $n > 1$ ,  $\frac{1}{n \ln(n)} < \frac{2}{n}$ , and the series  $2 \sum \frac{1}{n}$  diverges, so by the Comparison Test, the series  $\sum \frac{1}{n \ln(n)}$  diverges.
- 3. For all  $n > 1$ ,  $\frac{\ln(n)}{n^2} < \frac{1}{n^{1.5}}$ , and the series  $\sum \frac{1}{n^{1.5}}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.
- 4. For all  $n > 2$ ,  $\frac{n}{n^3-1} < \frac{2}{n^2}$ , and the series  $2 \sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{n}{n^3-1}$  converges.
- 5. For all  $n > 1$ ,  $\frac{\arctan(n)}{n^3} < \frac{\pi}{2n^3}$ , and the series  $\frac{\pi}{2} \sum \frac{1}{n^3}$  converges, so by the Comparison Test, the series  $\sum \frac{\arctan(n)}{n^3}$  converges.
- 6. For all  $n > 2$ ,  $\frac{\ln(n)}{n^2} > \frac{1}{n^2}$ , and the series  $\sum \frac{1}{n^2}$  converges, so by the Comparison Test, the series  $\sum \frac{\ln(n)}{n^2}$  converges.

**7.(1 pt)** The three series  $\sum A_n$ ,  $\sum B_n$ , and  $\sum C_n$  have terms

$$A_n = \frac{1}{n^9}, \quad B_n = \frac{1}{n^3}, \quad C_n = \frac{1}{n}.$$

Use the Limit Comparison Test to compare the following series to any of the above series. For each of the series below, you must enter two letters. The first is the letter (A,B, or C) of the series above that it can be legally compared to with the Limit Comparison Test. The second is C if the given series converges, or D if it diverges. So for instance, if you believe the series converges and can be compared with series C above, you would enter CC; or if you believe it diverges and can be compared with series A, you would enter AD.

- 1.  $\sum_{n=1}^{\infty} \frac{3n^3 + n^9}{935n^{12} + 7n^3 + 6}$
- 2.  $\sum_{n=1}^{\infty} \frac{3n^5 + n^2 - 3n}{7n^{14} - 6n^{11} + 5}$

- 3.  $\sum_{n=1}^{\infty} \frac{6n^2 + 3n^8}{3n^9 + 7n^3 - 3}$

**8.(1 pt)** The three series  $\sum A_n$ ,  $\sum B_n$ , and  $\sum C_n$  have terms

$$A_n = \frac{1}{n^6}, \quad B_n = \frac{1}{n^3}, \quad C_n = \frac{1}{n}.$$

Use the Limit Comparison Test to compare the following series to any of the above series. For each of the series below, you must enter two letters. The first is the letter (A,B, or C) of the series above that it can be legally compared to with the Limit Comparison Test. The second is C if the given series converges, or D if it diverges. So for instance, if you believe the series converges and can be compared with series C above, you would enter CC; or if you believe it diverges and can be compared with series A, you would enter AD.

- 1.  $\sum_{n=1}^{\infty} \frac{3n^3 + n^6}{374n^9 + 8n^3 + 6}$
- 2.  $\sum_{n=1}^{\infty} \frac{6n^2 + 7n^5}{3n^6 + 8n^3 - 3}$
- 3.  $\sum_{n=1}^{\infty} \frac{7n^5 + n^2 - 7n}{8n^{11} - 6n^8 + 2}$

**9.(1 pt)** Select the FIRST correct reason why the given series converges.

- A. Convergent geometric series  
 B. Convergent p series  
 C. Comparison (or Limit Comparison) with a geometric or p series  
 D. Converges by alternating series test

- 1.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(4n)}$
- 2.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+2}$
- 3.  $\sum_{n=1}^{\infty} \frac{(n+1)(3)^n}{2^{2n}}$
- 4.  $\sum_{n=1}^{\infty} \frac{n^2 + \sqrt{n}}{n^4 - 7}$
- 5.  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+7}$
- 6.  $\sum_{n=1}^{\infty} \frac{\sin^2(6n)}{n^2}$

**10.(1 pt)** Select the FIRST correct reason why the given series converges.

- A. Convergent geometric series  
 B. Convergent p series  
 C. Comparison (or Limit Comparison) with a geometric or p series  
 D. Cannot apply any test done so far in class

- 1.  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+4}$

- 2.  $\sum_{n=1}^{\infty} \frac{\sin^2(3n)}{n^2}$
- 3.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(6n)}$
- 4.  $\sum_{n=1}^{\infty} \frac{(n+1)(63)^n}{8^{2n}}$
- 5.  $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(e^n)}{n^7 \cos(n\pi)}$
- 6.  $\sum_{n=1}^{\infty} \frac{3(7)^n}{12^{2n}}$

11.(1 pt) Select the FIRST correct reason why the given series converges.

- A. Convergent geometric series
- B. Convergent p series
- C. Integral test
- D. Comparison with a convergent p series
- E. Converges by limit comparison test
- F. Converges by alternating series test

- 1.  $\sum_{n=1}^{\infty} \left(\frac{-e}{\pi}\right)^n$
- 2.  $\sum_{n=1}^{\infty} \frac{(n+1)(3^2-1)^n}{3^{2n}}$
- 3.  $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+7}$
- 4.  $\sum_{n=1}^{\infty} \frac{n^2 + \sqrt{n}}{n^4 - 7}$
- 5.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+5}$
- 6.  $\sum_{n=1}^{\infty} \frac{\sin^2(5n)}{n^2}$

12.(1 pt) Select the FIRST correct reason why the given series diverges.

- A. Diverges because the terms don't have limit zero
- B. Divergent geometric series
- C. Divergent p series
- D. Integral test
- E. Comparison with a divergent p series
- F. Diverges by limit comparison test
- G. Diverges by alternating series test

- 1.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(4)}$
- 2.  $\sum_{n=1}^{\infty} \frac{3n+4}{(-1)^n}$
- 3.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$
- 4.  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

- 5.  $\sum_{n=1}^{\infty} \frac{(n+1)(6^2+1)^n}{6^{2n}}$
- 6.  $\sum_{n=1}^{\infty} \frac{1}{n \ln(n)}$

13.(1 pt) A. Suppose that  $f(x)$  is a function that is positive and decreasing. Recall that by the integral test:

$$\int_p^{\infty} f(x)dx \leq \sum_{n=p}^{\infty} f(n).$$

Recall that  $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ . Suppose that for each positive integer  $k$ ,

$$f(k) = \frac{1}{k!}.$$

$$\int_2^{\infty} f(x)dx.$$

B = \_\_\_\_\_

B. A function is given by

$$h(k) = \int_0^{\infty} x^k e^{-x} dx.$$

. Its values may be found in tables. Make the change of variables  $y = x \ln(2)$  to express

$$I = \int_0^{\infty} x^4 2^{-x} dx$$

as a constant C times  $h(4)$ . Find C.

C = \_\_\_\_\_

C. Let  $g(x) = x^4 2^{-x}$ . Find the smallest number M such that the function  $g$  is decreasing for all  $x > M$ .

C. M = \_\_\_\_\_

D. Does  $\sum_{n=1}^{\infty} n^4 2^{-n}$  converge or diverge?

Answer with one letter, C or D.

14.(1 pt) For each sequence  $a_n$  find a number  $k$  such that  $n^k a_n$  has a finite non-zero limit.

(This is of use, because by the limit comparison test the series

$\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} n^{-k}$  both converge or both diverge.)

A.  $a_n = (5+4n)^{-7}$

k = \_\_\_\_\_

B.  $a_n = \frac{7}{n^2+n}$

k = \_\_\_\_\_

C.  $a_n = \frac{3n^2+7n+3}{6n^8+2n+4}$

k = \_\_\_\_\_

D.  $a_n = \left(\frac{3n^2+7n+5}{6n^8+2n+4\sqrt{n}}\right)^5$

k = \_\_\_\_\_

15.(1 pt) For each sequence  $a_n$  find a number  $r$  such that  $\frac{a_n}{r^n}$  has a finite non-zero limit.

(This is of use, because by the limit comparison test the series

$\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} r^n$  both converge or both diverge.)

A.  $a_n = (3+7^n)^{-5}$

r = \_\_\_\_\_

B.  $a_n = \frac{5^{2n}}{3^{n+n}}$

r = \_\_\_\_\_

C.  $a_n = \frac{5^n+n^5+5}{15^{8n}+3^n+7}$

$$r = \frac{5n^2 + 5n + 3^{0.8n}}{15n^8 + 3n + 7\sqrt{n}}$$

16.(1 pt) The series  $\sum_{n=1}^{\infty} n^k r^n$  converges when  $0 < r < 1$  and diverges when  $r > 1$ . This is true regardless of the value of the constant  $k$ . When  $r = 1$  the series is a p-series. It converges if  $k < -1$  and diverges otherwise. Each of the series below can be compared to a series of the form  $\sum_{n=1}^{\infty} n^k r^n$ . For each series determine the best value of  $r$  and decide whether the series converges.

A.  $\sum_{n=1}^{\infty} (4 + n(2)^n)^{-4}$

$r =$  \_\_\_\_\_ converges or diverges (c or d)? \_\_\_\_\_

B.  $\sum_{n=1}^{\infty} n^{\pi} \frac{4^{2n}}{7^n + n^9}$

$r =$  \_\_\_\_\_ converges or diverges (c or d)? \_\_\_\_\_

C.  $\sum_{n=1}^{\infty} \frac{n^5 + 4}{n^7 + 2}$

$r =$  \_\_\_\_\_ converges or diverges (c or d)? \_\_\_\_\_

D.  $\sum_{n=1}^{\infty} \left( \frac{4n^2 + 4n + 4^{-9n}}{7n^8 + 7n + 2\sqrt{n}} \right)^9$

$r =$  \_\_\_\_\_ converges or diverges (c or d)? \_\_\_\_\_

17.(1 pt) For each of the series below select the letter from a to c that best applies and the letter from d to k that best applies. A possible answer is af, for example.

- A. The series is absolutely convergent.
- B. The series converges, but not absolutely.
- C. The series diverges.
- D. The alternating series test shows the series converges.
- E. The series is a p-series.
- F. The series is a geometric series.
- G. We can decide whether this series converges by comparison with a p series.
- H. We can decide whether this series converges by comparison with a geometric series.
- I. Partial sums of the series telescope.
- J. The terms of the series do not have limit zero.
- K. None of the above reasons applies to the convergence or divergence of the series.

— 1.  $\sum_{n=1}^{\infty} \frac{3 + \sin(n)}{\sqrt{n}}$

— 2.  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

— 3.  $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\pi}$

— 4.  $\sum_{n=2}^{\infty} \frac{1}{n \log(2+n)}$

— 5.  $\sum_{n=1}^{\infty} \frac{(2n+4)!}{(n!)^2}$

— 6.  $\sum_{n=1}^{\infty} \frac{\cos^2(n\pi)}{n\pi}$

18.(1 pt)

For each of the series below select the letter from a to c that best applies and the letter from d to k that best applies. A possible correct answer is af, for example.

- A. The series is absolutely convergent.
- B. The series converges, but not absolutely.
- C. The series diverges.
- D. The alternating series test shows the series converges.
- E. The series is a p-series.
- F. The series is a geometric series.
- G. We can decide whether this series converges by comparison with a p series.
- H. We can decide whether this series converges by comparison with a geometric series.
- I. Partial sums of the series telescope.
- J. The terms of the series do not have limit zero.

— 1.  $\sum_{n=5}^{\infty} \frac{(3-1) \cdot ((2)3-1) \cdot \dots \cdot ((n-1)3-1)}{3^n (n!) \sqrt{n}}$

— 2.  $\sum_{n=1}^{\infty} \frac{n}{n+2^n}$

— 3.  $\sum_{n=1}^{\infty} (\log(n+1) - \log n)$

— 4.  $\sum_{n=1}^{\infty} (-1)^n \int_n^{n+1} 2^{-x} dx$

— 5.  $\sum_{n=1}^{\infty} \left( 1 + \frac{3}{n} \right)^n$

— 6.  $\sum_{n=1}^{\infty} \frac{n^6}{6^n}$

19.(1 pt) Select the FIRST correct reason why the given series diverges.

- A. Diverges because the terms don't have limit zero
- B. Divergent geometric series
- C. Divergent p series
- D. Integral test
- E. Comparison with a divergent p series
- F. Diverges by limit comparison test
- G. Cannot apply any test done so far in class

— 1.  $\sum_{n=1}^{\infty} \frac{(n+1)(10^2+1)^n}{10^{2n}}$

— 2.  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

— 3.  $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{(n!)^2}$

— 4.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

— 5.  $\sum_{n=1}^{\infty} (n)^{-\frac{1}{3}}$

— 6.  $\sum_{n=1}^{\infty} \frac{1}{n \ln(n)}$

Here is a short review of numerical series which you may find helpful.

REVIEW OF NUMERICAL SERIES  
SEQUENCES

A sequence is a list of real numbers. It is called convergent if it has a limit. An increasing sequence has a limit when it has an upper bound.

SERIES

(Geometric series, rational numbers as decimals, harmonic series, divergence test)

Given numbers forming a sequence  $a_1, a_2, \dots$ , let us define the  $n$ th partial sum as sum of the first  $n$  of them  $s_n = a_1 + \dots + a_n$ . The SERIES is convergent if the SEQUENCE  $s_1, s_2, s_3, \dots$  is. In other words it converges if the partial sums of the series approach a limit.

A necessary condition for the convergence of this SERIES is that  $a_n$ 's have limit 0. If this fails, the series diverges.

The harmonic series  $1+(1/2)+(1/3)+\dots$  diverges.

This illustrates that the terms  $a_n$  having limit zero does not guarantee the convergence of a series.

A series with positive terms, i.e.  $a_n > 0$  for all  $n$ , converges exactly when its partial sums have an upper bound.

The geometric series  $\sum_{n=1}^{\infty} r^n$  converges exactly when  $-1 < r < 1$ .

INTEGRAL AND COMPARISON TESTS

(Integral test, p-series, comparison tests for convergence and divergence, limit comparison test)

Integral test: Suppose  $f(x)$  is positive and DECREASING for all large enough  $x$ . Then the following are equivalent:

I.  $\int_1^{\infty} f(x)dx$  is finite, i.e. converges.

S.  $\sum_{n=1}^{\infty} f(n)$  is finite, i.e. converges.

This gives the p - test:  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges exactly when  $p > 1$ .

Comparison test: Suppose there is a fixed number  $K$  such that for all sufficiently large  $n$ :  $0 < a_n < K b_n$ .

Convergence. If  $\sum_{n=1}^{\infty} b_n$  converges then so does  $\sum_{n=1}^{\infty} a_n$ .

Divergence. If  $\sum_{n=1}^{\infty} a_n$  diverges then so does  $\sum_{n=1}^{\infty} b_n$ .

(Positive series having smaller terms are more likely to converge.)

Limit comparison test: SUPPOSE:  $a_n > 0, b_n > 0$  and  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = R$  exists. Moreover,  $R$  is not zero.

THEN  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$

both converge or both diverge.

OTHER CONVERGENCE TESTS FOR SERIES

(Alternating series test, absolute convergence, RATIO TEST)

Alternating series test: Suppose the sequence  $a_1, a_2, a_3, \dots$  is decreasing and has limit zero. Then  $\sum_{n=1}^{\infty} (-1)^n a_n$  converges.

This applies to  $(1)-(1/2)+(1/3)-(1/4)+\dots$

Absolute Convergence Test: IF  $\sum_{n=1}^{\infty} |a_n|$  converges,

THEN  $\sum_{n=1}^{\infty} a_n$  converges.

Ratio test:

SUPPOSE  $|\frac{a_{n+1}}{a_n}|$  has limit equal to  $r$ .

IF  $r < 1$  then  $\sum_{n=1}^{\infty} a_n$  CONVERGES.

IF  $r > 1$  the  $\sum_{n=1}^{\infty} a_n$  DIVERGES.