1.(1 pt) Find the interval of convergence for the given power series.

$$\sum_{n=1}^{\infty} \frac{(x-6)^n}{n(-8)^n}$$

The series is convergent
from $x =$ ________ , left end included (Y,N): ________
to $x =$ ________ , right end included(Y,N): ________

2.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(7x)^n}{n^8}$$

The series is convergent
from $x =$ ________ , left end included (Y,N): ________
to $x =$ ________ , right end included(Y,N): ________

3.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{(5)^n}$$

The series is convergent
from $x =$ ________ , left end included (Y,N): ________
to $x =$ ________ , right end included(Y,N): ________

4.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(3)^n (n^2+8)}$$

The series is convergent
from x = ________ , left end included (Y,N): ________
to x = ________ , right end included(Y,N): ________

5.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (2)^n x^n}{(\sqrt{n}+5)}$$

The series is convergent
from x = ________ , left end included (Y,N): ________
to x = ________ , right end included(Y,N): ________

6.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{x^n}{(4)^n (\sqrt{n}+11)}$$

The series is convergent
from x = ________ , left end included (Y,N): ________
to x = ________ , right end included(Y,N): ________

7.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x^n)(n+9)}{(10)^n}$$

The series is convergent
from x = ________ , left end included (Y,N): ________
to x = ________ , right end included(Y,N): ________

8.(1 pt) Find all the values of x such that the given series would converge.

$$\sum_{n=1}^{\infty} \frac{(6)^n (x^n)(n+1)}{(n+7)}$$

The series is convergent
from x = ________ , left end included (Y,N): ________
to x = ________ , right end included(Y,N): ________

9.(1 pt) Find the interval of convergence for the given power series.

$$\sum_{n=1}^{\infty} \frac{n^4 (x+1)^n}{(8^n)(n^{\frac{14}{3}})}$$

The series is convergent:
from $x =$ ________ , left end included (Y,N): ________
to $x =$ ________ , right end included (Y,N): ________

10.(1 pt) Match each of the power series with its interval of convergence.

— 1. $\sum_{n=1}^{\infty} \frac{(x-11)^n}{(n!)11^n}$
— 2. $\sum_{n=1}^{\infty} \frac{n!(4x-11)^n}{11^n}$
— 3. $\sum_{n=1}^{\infty} \frac{(4x)^n}{n^{11}}$
— 4. $\sum_{n=1}^{\infty} \frac{(x-11)^n}{(11)^n}$

A. $[\frac{-1}{4}, \frac{1}{4}]$
B. $(-\infty, \infty)$
C. $\{11/4\}$
D. $(0, 22)$

11.(1 pt) Suppose that $\frac{7x}{(13+x)} = \sum_{n=0}^{\infty} c_n x^n$.

Find the first few coefficients.
$c_0 =$ ________
$c_1 =$ ________
$c_2 =$ ________
$c_3 =$ ________
$c_4 =$ ________
Find the radius of convergence R of the power series.
$R =$ ________ .

12.(1 pt) The function $f(x) = \frac{10}{(1-3x)^2}$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the first few coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

13.(1 pt) The function $f(x) = \frac{7}{(1+7x)^2}$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the first few coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

14.(1 pt) The function $f(x) = \frac{1}{1+100x^2}$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the first few coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

15.(1 pt) The function $f(x) = 4x^2 \arctan(x^3)$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

What is the lowest term with a nonzero coefficient.

______

Find the radius of convergence R of the series.

$R =$ ______ .

16.(1 pt) The function $f(x) = 10x \arctan(8x)$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the first few coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

17.(1 pt) The function $f(x) = \ln(8-x)$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the first few coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

18.(1 pt) The function $f(x) = 8x\ln(1+x)$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the FOLLOWING coefficients in the power series.

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

$c_5 =$ ______

$c_6 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

19.(1 pt) The function $f(x) = 4x\ln(1+2x)$ is represented as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n.$$

Find the FOLLOWING coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

20.(1 pt) Represent the function $\frac{8}{(1-9x)}$ as a power series

$$f(x) = \sum_{n=0}^{\infty} c_n x^n$$

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence $R =$ ______ .

21.(1 pt) The function $f(x) = \ln(1-x^2)$ is represented as a power series

$f(x) = \sum_{n=0}^{\infty} c_n x^n$.

Find the FOLLOWING coefficients in the power series.

$c_0 =$ ______

$c_1 =$ ______

$c_2 =$ ______

$c_3 =$ ______

$c_4 =$ ______

Find the radius of convergence R of the series.

$R =$ ______ .

22.(1 pt) (a)

Evaluate the integral

$\int_0^2 \frac{48}{x^2+4}dx.$

Your answer should be in the form $k\pi$, where k is an integer. What is the value of k?

Hint: $\frac{d\arctan(x)}{dx} = \frac{1}{x^2+1}$

$k =$ ______

(b)

Now, lets evaluate the same integral using power series. First, find the power series for the function $f(x) = \frac{48}{x^2+4}$. Then, integrate it from 0 to 2, and call it S. S should be an infinite series. What are the first few terms of S ?

$a_0 =$ ______

$a_1 =$ ______

$a_2 =$ ______

$a_3 =$ ______

$a_4 =$ ______

(c) The answers to part (a) and (b) are equal (why?). Hence, if you divide your infinite series from (b) by k (the answer to (a)), you have found an estimate for the value of π in terms of an infinite series. Approximate the value of π by the first 5 terms.

______ .

(d)

What is the upper bound for your error of your estimate if you use the first 6 terms? (Use the alternating series estimation.)

______ .

23.(1 pt) Define the double factorial of n, denoted $n!!$, as follows:

$$n!! = \begin{cases} 1\cdot 3\cdot 5\cdots(n-2)\cdot n & \text{if n is odd} \\ 2\cdot 4\cdot 6\cdots(n-2)\cdot n & \text{if n is even} \end{cases}$$

where $(-1)!! = 0!! = 1$.

Find the radius of convergence for the given power series.

$$\sum_{n=1}^{\infty} \frac{9^n \cdot n! \cdot (3n+9)! \cdot (2n)!!}{3^n \cdot [(n+3)!]^3 \cdot (4n+3)!!}(2x+4)^n$$

The radius of convergence, R = ______

24.(1 pt) POWER SERIES AND TAYLOR POLYNOMIALS

Power Series

A power series $\sum_{n=0}^{\infty} a_n x^n$ has a RADIUS OF CONVERGENCE r.

The series converges for $|x| < r$ and diverges for $|x| > r$.

The radius of convergence is usually calculated by the ratio test, applied to the terms of the power series.

Suppose that $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|$ exists. Then the power series converges if

$|x| \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| < 1$ and diverges if $|x| \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| > 1$. The radius of convergence is $r = \left(\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}|\right)^{-1}$.

To determine whether the power series converges when $x = r$, replace x by r in the power series and decide whether the resulting numerical series, $\sum_{n=0}^{\infty} a_n r^n$ converges. The ratio test will not help in deciding this. Use some other convergence test.

To determine whether the power series converges when $x = -r$, proceed analogously.

Taylor and MacLaurin series

If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ converges in some interval $(-s,s)$ containing the point zero, then for each n :

$a_n = \frac{1}{n!} f^{(n)}(0)$.

Power series may be integrated or differentiated term by term.

That is:

$\frac{df}{dx} = \sum_{n=0}^{\infty} (n+1)a_{n+1}x^n.$

$\int_0^x f(t)dt = \sum_{n=1}^{\infty} \left(\frac{1}{n}\right) a_{n-1}x^n.$

The nth degree MacLaurin polynomial for f(x) is

$T_n(x) = \sum_{j=0}^{n} \frac{f^{(j)}(0)}{j!}x^j$

It approximates f(x) with error $R_n(x)$.

That is, $f(x) = T_n(x) + R_n(x)$. The size of the error is estimated by

$|R_n(x)| < M\frac{|x|^{(n+1)}}{(n+1)!}$.

Here, M is an upper bound for the (n+1)-st derivative of f between 0 and x. It is enough that

$|f^{(n+1)}(t)| < M$ for all t such that $|t| < |x|$.

For every statement above you should know the analogous statement for a power series in powers of $(x-c)$ which has the form $\sum_{n=0}^{\infty} a_n(x-c)^n$.

To receive a point enter the letter y.

answer ______